Ph 12b Quantum Physics
28 January 2010
Measurement

Note Title 1/28/2010

Our study of decoherence has indicated how a quantum system can exhibit classical behavior. Macroscopic quantum systems become correlated with the environment. If we observe only the system, and do not monitor the state of the environment, the state of the system appears to be *mixed.* A dust particle prepared in a coherent *superposition* of two different position eigensteates behaves like a probabilistic *ensemble* of two position eigenstates.

As you'll show in the homework, we can always choose orthogormal bases for the system S and its environment E so that the joint state vector of S and E can be expressed as

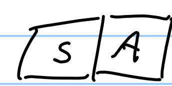
$$|\psi\rangle_{SE} = \sum_a \sqrt{p_a}\, |e_a\rangle_S \otimes |f_a\rangle_E,$$

where the density operator of the system is

$$\hat{\rho} = \sum_a p_a |e_a\rangle\langle e_a| .$$

Thus once the system becomes correlated with the environment, there is a preferred ON basis for the system, the basis in which its density operator is diagonal. In our discussion of phase damping, when $\Gamma t \gg 1$, this prefered basis is the position basis for the dust particle.

Now, how should we model the measurement of an observable? In a measurement, we bring a system S into contact with an apparatus A,



so that the system and apparatus interact, resulting in a correlation between system and apparatus.

E.g. suppose we want to measure S in the basis $\{|e_a\rangle_S\}$. Then we start the apparatus in a state $|g_0\rangle_A$, and couple S and A in such a way that a unitary transformation is induced:

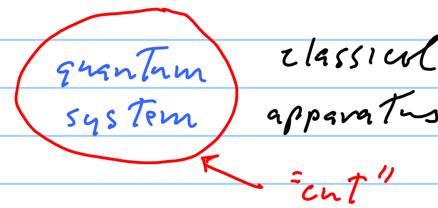
$$\hat{U}: |e_a\rangle_S \otimes |g_0\rangle_A \longrightarrow |e_a\rangle_S \otimes |g_a\rangle_S$$

or $\quad U: \left(\sum_a \psi_a |e_a\rangle_S\right) \otimes |g_0\rangle_A \longrightarrow \sum_a \psi_a |e_a\rangle_S \otimes |g_a\rangle_A$.

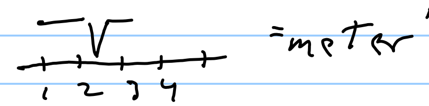
Now if we can readout the apparatus in the basis $\{|g_a\rangle\}$, we get the outcome $|g_a\rangle_A$ with probability $|\psi_a|^2$, and when we get this outcome, we have prepared the state $|e_a\rangle_S$ of the system.

But this explanation of measurement seems circular: we "explain" measurement of the system by assuming we can measure the apparatus.

In the traditional ("Copenhagen") discussion of measurement, we take it for granted that there is a "classical" world



exterior to the quantum system, and that the apparatus is "classical." The quantum system can exhibit interference, so it may not make sense to



say the state of the system is <u>either</u> $|e_0\rangle$ or $|e_1\rangle$; it may be in a coherent superposition of the two possibilities, like the electron passing through the double slit: there is no answer to the question "which way did the electron go." But classical systems do not interfere. Rather, there is a stable record of the reading on the meter we use to read out the apparatus, and we can come back tomorrow to check the reading: there <u>IS</u> a definite answer to the question: at which number on the dial is the meter pointing? In this point of view, in a quantum measurement we create a correlation between the microscopic quantum system and the macroscopic classical apparatus and perceiving the reading on the apparatus "collapses" the quantum system to an eigenstate of the observable being measured.

The theory of decoherence illuminates this distinction between the "quantum" and "classical" worlds. The apparatus behaves classically because it is strongly correlated with its unobserved environment. If we include the environment in our description, the state of system, apparatus, and environment becomes

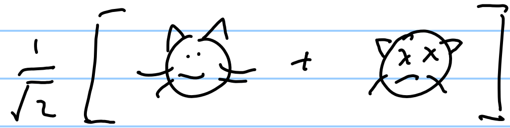
$$|\psi\rangle_{SAE} = \sum_a \psi_a |e_a\rangle_S \otimes |g_a\rangle_A \otimes |f_a\rangle_E$$

if the apparatus decoheres in the same basis as the basis in which we read it out. Ignoring the environment, then, the state of system and apparatus behaves like an ensemble, in which

$$|e_a\rangle_S \otimes |g_a\rangle_A \text{ occurs with probability } |\psi_a|^2$$

The "phase information" in the superposition $\sum_a \psi_a |e_a\rangle_S$ has been destroyed by decoherence. If the apparatus is truly "classical" information about the outcome $|e_a\rangle_S$ of the measurement is recorded robustly in the environment, so that interference between distinct outcomes cannot occur.

So decoherence explains why we may think of the apparatus as being in one of a set of definite states, in each of which the meter has a definite reading. But why is it that when I look at the apparatus, I perceive only <u>one</u> of these possible outcomes?



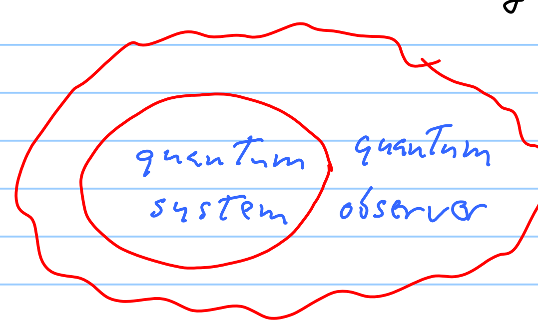
In Schrödinger's vivid example, we can imagine preparing a cat inside a box in a superposition of a dead state and a live state. Since the cat interacts with e.g. the air in the box, this state quickly evolves to

$$\frac{1}{\sqrt{2}}\left[\,|\text{Alive}\rangle_{cat}\otimes|A\rangle_E + |\text{Dead}\rangle_{cat}\otimes|D\rangle_E\,\right]$$

where $|A\rangle$ and $|D\rangle$ are very nearly orthogonal states of the environment. Ignoring the environment, we may argue that the cat is either dead or alive, each alternative occurring with probability 1/2.

But now I open the box and look inside and I perceive only one of these two alternatives. Why?

As a matter of pure logic, there is no paradox. If I want to explain what I see when I open the box, I should include myself



as part of the quantum system that the Schrödinger equation describes. When I look at the cat, I become correlated with its state — the state of the whole universe becomes

$$\frac{1}{\sqrt{2}}\Big(|\text{dead}\rangle_{cat} \otimes |D\rangle_E \otimes |\text{I know it's dead}\rangle_{me}$$

$$+ |\text{alive}\rangle_{cat} \otimes |A\rangle_E \otimes |\text{I know it's alive}\rangle_{me} \Big)$$

This state encompasses both alternative outcomes of my "measurement" of the cat. Yet, for either outcome, I believe there was a <u>definite</u> outcome. The probability is <u>zero</u> that I perceive the cat as being half alive and half dead. Yet this process could be explained invoking <u>only</u> unitary evolution — there is no need to hypothesize a physical process causing the state vector to collapse. (But I pay the rather heavy price of including myself in the system the theory describes!)

Suppose when I look at the cat I find it alive. Then I am no longer interested in the other alternative (dead) that might have happended, because it did not happen. To predict what will happen next to the cat, as I will perceive it, I may assume that its state is now $|\text{Alive}\rangle_{cat}$. This is the "collapse" — it occurs not as a fundamentol process, instead I do it for convenience, taking into account what I <u>learned</u> in the measurement.

The theory describes how what I see at time $T'+t$ is correlated with what I see at time T', and once I <u>know</u> what I see at time T', I can forget about the alternative things I might have seen.

But, if the state of the system is $\sum_a \psi_a |e_a\rangle_S$, why should $|\psi_a|^2$ be the <u>probability</u> that outcome $|\psi_a|^2$ will be found when I measure?

First consider the case where N <u>decohering</u> alternatives all have equal amplitudes,

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_a |e_a\rangle_S \otimes |f_a\rangle_E$$

when I am about to look at the system, it is natural that I assign the same probability to all the alternatives $\{|e_a\rangle\}$ because nothing distinguishes one alternative from the others.

For example, $\quad |\psi\rangle = \frac{1}{\sqrt{2}}\left(|e_0\rangle_S \otimes |f_0\rangle_E + |e_1\rangle_S \otimes |f_1\rangle_E\right),$

where the states $|f_0\rangle_E$ and $|f_1\rangle_E$ are unobserved, ought to be equivalent from my perspective to the state $\quad |\psi'\rangle = \frac{1}{\sqrt{2}}\left(|e_0\rangle_S \otimes |f_1\rangle_E + |e_1\rangle_S \otimes |f_0\rangle_E\right).$

If the state of the environment is not observed, all we care about is that $|f_0\rangle_E$ and $|f_1\rangle_E$ are orthogonal, not what these states are. But $|\psi'\rangle$ can be obtained from $|\psi\rangle$ by swapping the two states of the system $|e_0\rangle_S \longleftrightarrow |e_1\rangle_S$.

This swap, then, leaves unchanged my description of the system, so I assign equal probabilities to both.

But now consider a more general superposition

$$|\psi\rangle_S = \sqrt{\frac{p}{q}}\,|e_0\rangle_S + \sqrt{\frac{q-p}{q}}\,|e_1\rangle_S$$

where p, q are positive integers with $p < q$

The state decoheres:

$$|\psi\rangle = \sqrt{\frac{p}{q}}\,|e_0\rangle_S \otimes |f_0\rangle_E + \sqrt{\frac{q-p}{q}}\,|e_1\rangle_S \otimes |f_1\rangle_E$$

But we could choose a different basis for the environment, so that

$$|f_0\rangle_E = \frac{1}{\sqrt{p}} \sum_{a=1}^{p} |f_a'\rangle_E \qquad |f_1\rangle_E = \frac{1}{\sqrt{q-p}} \sum_{a=p+1}^{q} |f_a'\rangle_E$$

so that

$$|\psi\rangle = \frac{1}{\sqrt{q}}\left(\sum_{a=1}^{p} |e_0\rangle_S \otimes |f_a'\rangle_E + \sum_{a=p+1}^{q} |e_1\rangle_S \otimes |f_a'\rangle_E \right)$$

Now this is a uniform superposition. We can <u>imagine</u> that E interacts with <u>another</u> environment G and becomes correlated with <u>it</u>:

$$|f_a'\rangle_E \otimes |g_0\rangle_G \longrightarrow |f_a'\rangle_E \otimes |g_a\rangle_G$$

This imaginary interaction between environments can't change how we perceive the system

Now

$$|\Psi\rangle = \frac{1}{\sqrt{q}}\left(\sum_{a=1}^{p} |e_0\rangle_S \otimes |f_a'\rangle_E \otimes |g_a\rangle_G + \sum_{a=p+1}^{q} |e_1\rangle_S \otimes |f_a'\rangle_E \otimes |g_a\rangle_G\right)$$

This can be viewed as a state in which the alternatives

$$\{ |e_0\rangle_S \otimes |f_a'\rangle_E,\ a=1,\dots,p\} \cup \{|e_1\rangle_S \otimes |f_a'\rangle,\ a=p+1,\dots,q\}$$

have all strongly decohered, and occur with equal amplitude, so that each should be assigned probability $1/q$.

Hence $P(e_0) = \sum_{a=1}^{p} \frac{1}{q} = p/q$, $P(e_1) = \sum_{a=p+1}^{q} \frac{1}{q} = \frac{q-p}{q}$.

Therefore, we recover the usual probability rule $|\psi\rangle_S = \psi_0 |e_0\rangle + \psi_1 |e_1\rangle \Longrightarrow$

$$P(e_0) = |\psi_0|^2 \text{ and } P(e_1) = |\psi_1|^2,$$

at least for the case where the probabilities are rational numbers.

A further question is: If decoherence picks out a prefered basis for the system, what determines that basis? E.g. after the dust particle interacts with the environment, the joint state of S and E is

$$|\Psi\rangle_{SE} = \sum_a \sqrt{p_a}\, |e_a\rangle_S \otimes |f_a\rangle_E$$

where the $\{|e_a\rangle_S\}$ are states that have a definite <u>position</u> and the states $|f_a\rangle_E$ are corresponding macroscopic states (i.e. the recording of position information in the environment is highly redundant). Why does the system become an ensemble of possible position eigenstates rather than an ensemble of, say, momentum eigenstates?

This distinction between position and momentum must have something to do with the Hamiltonian of the world, in particular the nature of interaction between system and environment. Such interactions are typically local in space (degrees of freedom interact with other degrees of freedom in close spatial proximity). As a result, the position of the system, rather than its momentum, is what gets imprinted on the environment.