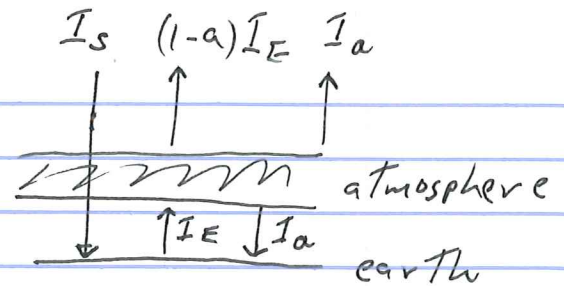


Global Warming

Sunlight passes through atmosphere, intensity I_s absorbed by earth (reflected light not included in I_s).



Earth radiates in IR with intensity I_E ; a fraction a of this IR radiation is absorbed by atmosphere.

Atmosphere radiates intensity I_a both into space and down to earth. Earth absorbs radiation from atm.

Earth in equilibrium: $I_E = I_s + I_a$
(absorbed intensity matches radiated intensity).

Intensity emerging from atm matches intensity incident from sun:

$$I_s = I_a + (1-a)I_E$$

Eliminate I_a : $I_s = (I_E - I_s) + (1-a)I_E = (2-a)I_E - I_s$
 $\Rightarrow I_E = \left(\frac{2}{2-a}\right) I_s$

Limiting cases: $a=0 \Rightarrow I_E = I_s$ (earth radiates at rate it absorbs.)

$a=1 \Rightarrow I_E = 2I_s$ (Now radiation from atm matches solar intensity. On earth, equal intensity from sun and atm.)

a is increasing as CO_2 in atm increases $\Rightarrow$ earth warms. We know

$$a \approx .75, \quad a_{\text{CO}_2} \approx .07, \quad a_{\text{CO}_2} \propto \text{concentration } n_{\text{CO}_2}$$

$$n = 400 \text{ ppm}$$

$$\Delta n = 2 \text{ ppm/yr} \Rightarrow \Delta T_E \sim .02^\circ/\text{yr}$$

Why? $I_E = \sigma_B T_E^4 \Rightarrow T_E \propto \left(\frac{2}{2-a}\right)^{\frac{1}{4}}$

$$\Rightarrow \Delta T_E = \Delta a \frac{1}{4} (2-a)^{-1} T_E \approx \frac{1}{5} \Delta a T_E$$

($a \approx .75$)

$$\Delta a_{\text{CO}_2} = a_{\text{CO}_2} (\Delta a/a)_{\text{CO}_2}$$

$$= .07 \frac{2 \text{ ppm}}{400 \text{ ppm}}$$

$$\Rightarrow \Delta T_E = \frac{1}{5} (.07) \frac{2}{400} (T_E \approx 300^\circ \text{K})$$

$$\approx (7 \times 10^{-5}) (300) \approx .02^\circ/\text{yr} (^\circ \text{C})$$

Earth warms 1°C in 50 years, assuming $\Delta n = 2 \text{ ppm/year}$
and $a_{\text{CO}_2} \propto n_{\text{CO}_2}$.

(of course, model is vastly oversimplified...)