Remark on history
GR was formal + sterile until
- Penrose, Hawking, Kerr
- μwave BG

Cosmology

We are in an exciting era:
- Relativistic astrophysics, e.g. γ bursts and BH's or NS's
- LIGO - a new window
- Cosmology at the cross roads

⇓ A particularly good topic for this course

combines: [hydrodynamics, plasma physics, optics, relativity, ...

⇙

we should discuss μwave anisotropy, and what we can expect to learn from it!

The assumptions of relativistic cosmology

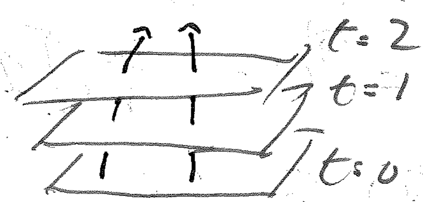
[① Homogeneity ② Isotropy] (Copernicus)

③ Einstein Eqn

→ A reasonable caricature of observed universe at large scales

→ What do these assumptions mean?

We can choose three-dimensional hypersurfaces on which the spatial metric ${}^{(3)}ds^2$



has 6-parameter group of isometries: "translations" and "rotations" that leave metric invariant

["rotations" leave a point fixed.]

use as a time coordinate: proper time measured by the "comoving" (freely falling) galaxies $\Rightarrow$ homogeneous and isotropic at each value of t

If we assume isotropy about a fixed point

$$ds^2 = -dt^2 + a^2(t)\left[\underbrace{e^{2\Lambda(r)}}_{= g_{rr}(r)} dr^2 + r^2 d\Omega^2\right]$$

- a^2 is a function of t only = scale factor
- $e^{2\Lambda(r)}$ is a function of r to be determined

$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ - required by spherical symmetry

- r defined by e.g. circumference of sphere $C = 2\pi r a$ (not by proper distance to sphere from $r=0$)

Now we demand homogeneity, which means, in particular, that scalar curvature must be a constant (on 3 dim hypersurface)

Exercise: compute it (with a^2 scaled out)

$$R^i_{\ i} = {}^{(3)}R = \frac{2}{r^2}\left[1 - \frac{d}{dr}\left(r e^{-2\Lambda}\right)\right] = \text{constant} \equiv 6K$$

Integrate this

$$1 - 3Kr^2 = \frac{d}{dr}\left(r e^{-2\Lambda}\right) \Rightarrow r - Kr^3 = r e^{-2\Lambda} + C$$

$$\Rightarrow e^{-2\Lambda} = 1 - Kr^2 - C/r$$

$$\Rightarrow \quad g_{rr}(r) = e^{2\Lambda} = \frac{1}{1-Kr^2 - C/r}$$

To fix constant of integration C:

r defined by
circum $= 2\pi r$, and
geometry is locally flat at $r=0$

So r does coincide with flat space radial coord at $r=0$ $\Rightarrow$ $g_{rr}(r=0) = 1$ $\Rightarrow$ $C=0$

$$\boxed{g_{rr}(r) = \frac{1}{1-Kr^2}} \quad \text{and} \quad \boxed{6K = {}^{(3)}R}$$

or $${}^{(3)}R = \frac{6K}{a^2(t)}$$

And ~~now~~ now we have

$$\cancel{ds^2 = -dt^2 \left[\frac{dr^2}{1-K}\right]}$$

We can rescale r, and choose scale factor so that

$K = 1, 0, -1$

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$$

$K = 0, 1, -1$

So far, we have not yet shown that the ${}^{(3)}ds^2$ is really homogeneous — but it is!

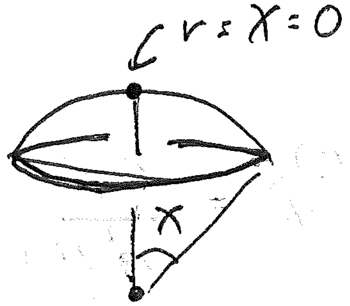
① $K=0$ ("flat") $\quad {}^{(3)}ds^2 = a^2[dr^2 + r^2 d\Omega^2]$

obviously just 3-dim Euclidean space (and has ∞ volume)

② $K=1$ $\quad {}^{(3)}ds^2 = a^2\left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2\right]$

This is the metric of a 3-sphere

Let $d\chi^2 = \frac{dr^2}{1-r^2}$ or $d\chi = \frac{dr}{\sqrt{1-r^2}}$



$r = \sin\xi \Rightarrow d\chi = d\xi$ or $\chi = \xi + \text{const}$

$${}^{(3)}ds^2 = a^2\left[d\chi^2 + \sin^2\chi\,({}^{(2)}d\Omega^2)\right]$$
$$r = \sin\chi$$

Visualize as a 3-sphere embedded in 4-dim space

r = "radius" of 2-sphere
a = radius of 3-sphere

All points on a 3 sphere are equivalent $\Rightarrow$ homogeneous (rotations act transitively)

Note: total volume $2\pi^2 a^3$ is finite
= "closed universe"

③ $K=-1$

$${}^{(3)}ds^2 = a^2\left[\frac{dr^2}{1+r^2} + r^2 d\Omega^2\right]$$

$$= a^2\left[d\chi^2 + \sinh^2\chi\,({}^{(2)}d\Omega^2)\right]$$
$$r = \sinh\chi$$

= "open" universe
whose spatial slices are hyperboloids

E.g. in case (2)

$${}^{(4)}ds^2 = dw^2 + dx^2 + dy^2 + dz^2$$

let

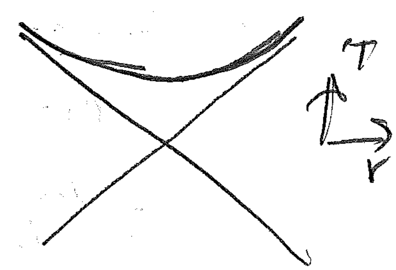
$$w = R\cos\chi$$
$$z = \cancel{R\cos\chi}\ R\sin\chi\cos\theta$$
$$x = R\sin\chi\sin\theta\cos\phi \quad \Rightarrow \quad {}^{(4)}ds^2 = dR^2 + R^2\,{}^{(3)}ds^2$$
$$y = R\sin\chi\sin\theta\sin\phi$$

<u>What about case (3)</u>?

We can't embed in 4-dim Euclidean space

$$C_{2sphere} = 2\pi\sinh\chi \leftarrow \text{grows exponentially}$$

Instead — embed in ${}^{(4)}$Minkowski as surface of fixed <u>Lorentzian</u> distance from the origin



$$T = a\cosh\chi \qquad (T^2 - r^2 = a^2)$$
$$r = a\sinh\chi$$

$$-dT^2 + dr^2 = a^2(\cosh^2\chi - \sinh^2\chi)\,d\chi^2 = a^2\,d\chi^2$$

Homogeneous, because <u>Lorentz</u> transformations act transitively on the hyperboloid

Now we have found <u>all</u> homogeneous and isotropic 3-geometries

Comment $\Gamma^\mu_{tt} = \frac{1}{2} g^{\mu\sigma}(2g_{t\sigma,t} - g_{tt,\sigma}) = 0$ So $(r,\theta,\phi) = \text{const}$ <u>is</u> a geodesic i.e. $\frac{\partial}{\partial t}$

Dynamics of the universe:

So far, our analysis is merely kinematic
Einstein eqn determines how $a(t)$ evolves

To solve $\boxed{\begin{gathered} G_{\mu\nu} = 8\pi T_{\mu\nu} \\ T^{\mu\nu}{}_{;\nu} = 0 \end{gathered}}$ $\Leftarrow$ what goes on the RHS of eqn? Must be homogeneous and isotropic

$T^{0i} = 0$ (No momentum density)

in local flat coords $\left[T^i{}_j \propto \delta^i{}_j \right.$ $T^{\mu\nu} = \mathrm{diag}(\rho, P, P, P)$ $\Leftarrow$ Has this form in "Rest frame" of the fluid

a "perfect fluid" with ρ = energy density
P = (isotropic) pressure

- Conservation of energy momentum

$$0 = T^{\mu\nu}{}_{;\nu}$$

$$T^{\mu\nu}{}_{;\lambda} = \partial_\lambda T^{\mu\nu} + \Gamma^\mu_{\lambda\sigma} T^{\sigma\nu} + \Gamma^\nu_{\lambda\sigma} T^{\mu\sigma}$$

$$0 = T^{\mu\nu}{}_{;\nu} = \partial_\nu T^{\mu\nu} + \Gamma^\mu_{\nu\sigma} T^{\nu\sigma} + \Gamma^\nu_{\nu\sigma} T^{\mu\sigma}$$

Because of spatial homogeneity, only $\mu = 0$ is informative

$$0 = \partial_0 T^{00} + \Gamma^0_{\nu\sigma} T^{\nu\sigma} + \Gamma^\nu_{\nu 0} T^{00}$$

$$\Gamma^0_{\nu\sigma} = \frac{1}{2} g^{0\lambda} \left[g_{\nu\lambda,\sigma} + g_{\sigma\lambda,\nu} - g_{\nu\sigma,\lambda} \right]$$

$$= -\frac{1}{2} \left[g_{\nu 0,\sigma} + g_{\sigma 0,\nu} - g_{\nu\sigma,0} \right]$$

$\Gamma^0_{ij} = +\frac{1}{2} g_{ij,0} =$ and $g_{ij,0} = +2\frac{\dot a}{a} g_{ij}$

$$\Rightarrow \Gamma^0_{ij} T^{ij} = +3 \frac{\dot a}{a} P$$

And we also have

$$\Gamma^{\nu}_{\nu 0} = \frac{1}{2} g^{\nu\eta}\left[g_{\nu\eta,0} \right] = \frac{1}{2} g^{ij} g_{ij,0} = 3\frac{\dot a}{a}$$

Hence $$0 = \dot\rho + 3\frac{\dot a}{a}p + \frac{3\dot a}{a}\rho$$

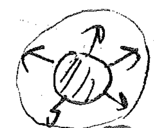
Or -- $$\frac{d}{dt}(\rho a^3) = -p\frac{d}{dt}(a^3)$$

Two limiting cases:

① $p = 0 \Rightarrow \rho a^3 = \text{const}$

② $p = \frac{1}{3}\rho \Rightarrow \rho a^4 = \text{const}$

This is easy to interpret



The volume of an element of comoving fluid is $\propto a^3$

above is $dU = -p\,dV$

case ② – redshift from stretching of λ.

(The universe is close to ① or ② for most of its history)

Next: Einstein eqn: $G_{00} = 8\pi T_{00}$

Can evaluate (exercise)

$$G_{00} = 3\left(\frac{\dot a}{a}\right)^2 + 3\frac{K}{a^2} = 8\pi\rho$$

or $$\boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}\rho - \frac{K}{a^2}}$$

(and the other components give no independent info –)

what is interpretation here?

collapsing sphere

Newtonian spherical shell of radius r $\quad \ddot r = -\frac{GM}{r^2}$

or energy conservation $\quad \frac{1}{2}\dot r^2 + \left(-\frac{GM}{r}\right) = \text{constant}$

Isotropic collapse

$$M = \frac{4\pi}{3}\rho R^3 \Rightarrow \frac{1}{2}\dot{r}^2 + \left(-\frac{4\pi}{3}\rho r^2\right) = \text{const}$$

$$\Rightarrow \boxed{\frac{\dot{r}^2}{r^2} = \frac{8\pi}{3}\rho + \frac{\text{const}}{r^2}}$$

All we have added is a geometric interp of the constant

An eqn Newton could have derived -- But he made a mistake -- he said that ∞ homogeneous universe would not collapse -- But in fact it can, with isotropy about each point preserved

Evolution of universe:

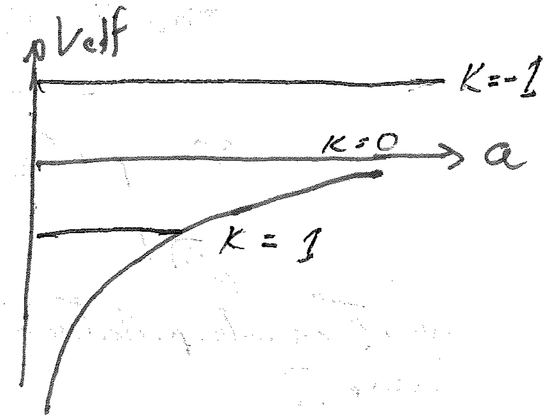
$$\frac{1}{2}\dot{a}^2 + \underbrace{\left(-\frac{4\pi}{3}\rho a^2\right)}_{\text{Effective potential}} = \underbrace{-\frac{1}{2}K}_{\text{Total energy}}$$

$$A = \frac{8\pi}{3}\rho a^3$$

E.g. Matter dominated $\Rightarrow \frac{4\pi}{3}\rho a^2 = \frac{\frac{1}{2}A}{a}$

$$\Rightarrow \frac{1}{2}\dot{a}^2 - \frac{1}{2}\frac{A}{a} = -\frac{1}{2}K$$

$$\Rightarrow \frac{da}{dt} = \sqrt{\frac{A}{a} - K}$$



① Consider flat ($K=0$)

$$\Rightarrow a^{\frac{1}{2}}da = \sqrt{A}\,t \Rightarrow \frac{2}{3}a^{3/2} = \sqrt{A}\,(t + \text{const})$$

$$\Rightarrow a = \left[\tfrac{3}{2}\sqrt{A}\,t\right]^{2/3}$$

(if $a=0$ at $t=0$)

For $K = \pm 1$ — useful to introduce a new time coordinate

$$d\eta = \frac{dt}{a} \quad \text{"conformal time"}$$

More about η later on

or $\dfrac{d}{dt} = \dfrac{1}{a}\dfrac{d}{d\eta}$

So $\quad a^{\frac{1}{2}} \dfrac{da}{dt} = \sqrt{A - Ka}$

$$= a^{-\frac{1}{2}} \frac{da}{d\eta} = 2\frac{d}{d\eta} a^{\frac{1}{2}} = \sqrt{A - Ka}$$

Hence $\boxed{\frac{1}{2}\eta = \int \frac{dx}{\sqrt{A - Kx^2}}}$ $\quad x = \sqrt{a}$

and $t = \int a(\eta)\, d\eta$

$K = 1 \quad x = \sqrt{A}\sin\theta \Rightarrow \frac{1}{2}\eta = \theta + \text{const}$

$$= \arcsin\left[\sqrt{\frac{a}{A}}\right]$$

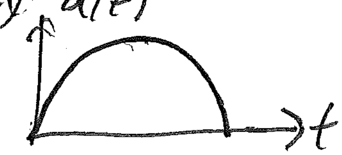
$$\Rightarrow \sqrt{\frac{a}{A}} = \sin\tfrac{1}{2}\eta \Rightarrow \frac{a}{A} = \sin^2\tfrac{1}{2}\eta = \tfrac{1}{2}(1 - \cos\eta)$$

$$\Rightarrow \boxed{a = \tfrac{1}{2}A(1 - \cos\eta)}$$

and $t = \int a(\eta)\,d\eta \Rightarrow \boxed{t = \tfrac{1}{2}A(\eta - \sin\eta)}$

$\Rightarrow$

These are parametric eqns of a cycloid



Universe recollapses at $\eta = 2\pi$

$t = \pi A$

And in the $K=-1$ case:

$$a = \tfrac{1}{2} A (\cosh \eta - 1)$$
$$t = \tfrac{1}{2} A (\sinh \eta - \eta)$$

Now universe expands forever

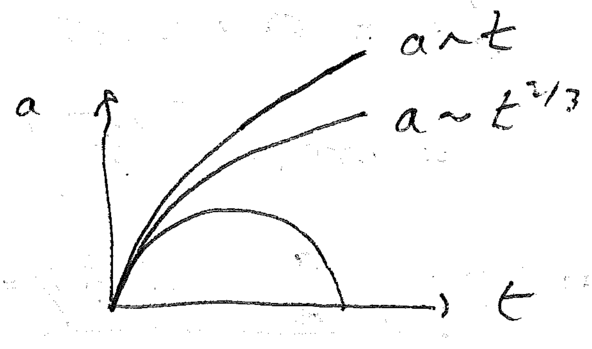
NOTE: in both the $K=1$ and $K=-1$ universes

for η small we have $a = \tfrac{1}{2} A (\tfrac{1}{2}\eta^2) + \dots$
$t = \tfrac{1}{2} A (\tfrac{1}{6}\eta^3) + \dots$

$$\Rightarrow \eta \sim \left(\frac{12}{A} t\right)^{1/3}$$

$$a = \frac{A}{4}\left(\frac{144 t^2}{A^2}\right)^{1/3} = \left(\frac{144}{64}\right)^{\frac{1}{3}} A^{\frac{1}{3}} t^{2/3} = \left(\tfrac{3}{2}\sqrt{A}\, t\right)^{2/3}$$

— the flat space answer — [curvature is unimportant early in the history of the universe (i.e. univ. does not yet "know" that it is curved.]



Why $a \sim t$? This is the $\rho = 0$ soln

Is $ds^2 = -dt^2 + t^2 \left[\frac{dr^2}{1+r^2} + r^2 d\Omega^2\right]$ the same as flat Minkowski spacetime (It is actually static)

$\Uparrow$ Here $K=-1$

recall our embedding:
$r = a \sinh \chi$
$T = a \cosh \chi$

- No $\rho = 0$ soln for $K=+1$
- $\rho = 0$ and $K=0 \Rightarrow a = \text{const}$

In the metric
$$ds^2 = -dt^2 + t^2\left[d\chi^2 + \sinh^2\chi\, d\Omega^2\right]$$
we may choose
$$r = t\sinh\chi$$
$$T = t\cosh\chi$$
$$\Rightarrow -dT^2 + dr^2 = -(dt\cosh\chi + d\chi\, t\sinh\chi)^2 + (dt\sinh\chi + d\chi\, t\cosh\chi)^2$$
$$= -dt^2 + t^2 d\chi^2 \quad \checkmark$$

Expanding universe

What does it mean that universe is expanding?

Proper distance between comoving galaxies is increasing (but so what)?

$$d_{prop} = \int_0^r dr\sqrt{g_{rr}} = a(t)\int \frac{dr}{\sqrt{1-Kr^2}} = a(t)\,\chi$$

Distance along geodesic on a time slice

$$\boxed{\chi = \begin{cases} \sin^{-1} r & K = 1 \\ r & 0 \\ \sinh^{-1} r & -1 \end{cases}}$$

(Is $t, \theta, \varphi =$ const a geodesic?

$$\Gamma^\mu_{rr} = \tfrac{1}{2} g^{\mu\lambda}\left[2g_{r\lambda,r} - g_{rr,\lambda}\right]$$

$$\Gamma^0_{rr} = +\tfrac{1}{2} g_{rr,0} = +\tfrac{1}{2}\frac{2\dot a}{a} g_{rr} = \frac{\dot a}{a} g_{rr}$$

[So – it is a geodesic of the 3 geometry, but not the 4 geometry!])

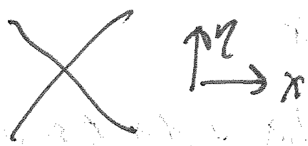
A better question: Consider a null geodesic

Note: you can't "see the back of your head" in closed universe. Recollapse at $\eta = 2\pi$ $\Rightarrow$ $\chi = 2\pi$ (Once around: North pole to North pole)

For a radial null geodesic

$$ds^2 = 0 = -dt^2 + a(t)^2 d\chi^2$$
$$= a(t)^2\left[-d\eta^2 + d\chi^2\right]$$

i.e. LC's are 45° angles



$$d\eta = \frac{dt}{a(t)}$$

η = "conformal time" because metric in this form is conformal to flat space (differs by $g_{\mu\nu} = a^2 g_{\mu\nu}^{(flat)}$)

Hence a geodesic "travels" to where

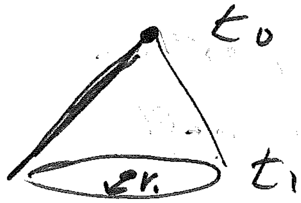
$$\boxed{\begin{array}{c} \chi = r_1 \\ \sinh\chi = r_1 \\ \sin\chi = r_1 \end{array}}$$

$$\boxed{\chi = \eta(t_0) - \eta(t_1)}$$

and $\Delta\eta = \int_{t_1}^{t_0} \frac{dt}{a(t)}$ (emitted at t_1 from r_1, received at t_0 at $r = 0$)

$$r = \Sigma(\chi) = \begin{array}{c} \sin\chi \\ \chi \\ \sinh\chi \end{array} \Rightarrow r = \Sigma(\eta_0 - \eta_1)$$
$$= \Sigma(\eta_0)\,C(\eta_1) - \Sigma(\eta_1)\,C(\eta_0)$$
$$C(\eta) = \begin{array}{c} \cos\eta \\ 1 \\ \cosh\eta \end{array}$$

Consider comoving observers



$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \chi \quad \text{is time independent}$$

So suppose signals are sent proper time δt_1 apart; they are received δt_0 apart where

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)} = \int_{t_1}^{t_0} \frac{dt}{a(t)} \Rightarrow \frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)}$$

and hence $\dfrac{\nu_0}{\nu_1} = \dfrac{a(t_1)}{a(t_0)}$

$$\frac{\nu_1}{\nu_0} \equiv 1+z = \frac{a(t_0)}{a(t_1)}$$

For small z, we may interpret this as doppler shift due to Hubble flow

Define $H_0 = \dfrac{\dot{a}_0}{a_0}$ $\quad q_0 = \dfrac{-a_0\ddot{a}_0}{\dot{a}_0^2} \Rightarrow H_0^2 q_0 = a_0^{-1}\ddot{a}_0$

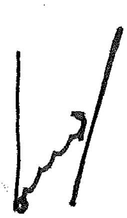
Expand $a(t) = a_0\left[1 - H_0(t_0-t) - \frac{1}{2} q_0 H_0^2 (t_0-t)^2\right]$ $+ -$

Since $\bullet \longleftrightarrow \bullet$

proper distance between comoving objects is $d = a(t)L$, distance increases at a rate

$$V = \dot{d} = \dot{a}L = \frac{\dot{a}}{a}d = Hd$$

$H_0 = \dfrac{\dot{a}_0}{a_0}$ is Hubble constant



Receding at velocity $v \Rightarrow$

$$1+z \equiv \frac{\nu_1}{\nu_0} \simeq 1+v \Rightarrow z \sim v$$

Agrees with $1+z = \dfrac{a_0}{a_1} \sim 1 + H_0\Delta t$

$$z \sim H_0 \Delta t \sim H_0 d \sim v$$

But -- unless $z \ll 1$, it is not particularly useful to describe z in terms of a recession velocity

---

z increases with distance, but distance is hard to measure -- and it is ambiguous, because universe is exchanging during exchange of light pulses

2 ways:

① Angular diameter (eg. apparent angular size of a galaxy)

$$d_A = \frac{D}{\delta} = \frac{\text{Proper diameter}}{\text{Angular size}}$$

use metric:

$$D = a(t_1) r_1 \delta$$

But $\delta = \dfrac{D}{r_1 a(t_1)}$

(i.e. circumference is $2\pi r_1 a(t_1)$ at time of emission

$$\Rightarrow \boxed{d_A = r_1 a(t_1)}$$

~~② Proper motion (eg. expansion of exploding quasar)~~

$\cancel{d_M = \frac{v}{\delta}}$

~~Because~~ $\cancel{v = \frac{d}{dt}\left[R(t_1) r_1 \delta\right]}$

② Luminosity distance $\quad l = \dfrac{L}{4\pi d_L^2}$ (assumed 150 Kpc)

$$d_L^2 = \frac{L}{4\pi l}$$

L = luminosity of source

l = power/area detected

Useful Formulae

we can use some algebraic tricks to express e.g. scale factor, coordinate r_1 of an emitted photon, luminosity distance of a source, in terms of q_0, H_0, z

① Can express ρ in terms of H and q

$$\dot{a}^2 = \frac{8\pi}{3}\rho a^2 - K \Rightarrow H = \frac{8\pi}{3}\rho - \frac{K}{a^2}$$

differentiate $\quad 2\dot{a}\ddot{a} = \frac{8\pi}{3}\frac{d}{dt}(\rho a^2)$

Assume: matter dominated $\rho \sim \frac{1}{a^3}$

Hence $\quad \ddot{a} = -\frac{4\pi}{3}\rho a$

$$\frac{d}{dt}\rho a^2 = -\frac{\dot{a}}{a}\rho a^2$$

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{4\pi}{3}\rho\frac{a^2}{\dot{a}^2} \Rightarrow \boxed{\frac{4\pi}{3}\rho = H^2 q}$$

Warning: matter dominated

If $K=0 \quad H^2 = \frac{8\pi}{3}\rho_c \leftarrow$ critical density

$$\Rightarrow q = \frac{1}{2}\frac{\rho}{\rho_c} \equiv \frac{1}{2}\Omega$$

Hence
$q_0 > \frac{1}{2} \quad (\Omega_0 > 1) \Rightarrow$ closed
$q_0 = \frac{1}{2} \quad (\Omega_0 = 1) \Rightarrow$ flat
$q_0 < \frac{1}{2} \quad (\Omega_0 < 1) \Rightarrow$ open

② Express a in terms of H and q

$$H^2 = \frac{8\pi}{3}\rho - \frac{K}{a^2} \Rightarrow \frac{K}{a^2} = \frac{8\pi}{3}\rho - H^2 = 2H^2 q - H^2 = H^2(2q-1)$$

$$\Rightarrow a^2 = \frac{K}{H^2\, 2q-1} = \frac{1}{H^2(2q-1)} \quad (K \neq 0)$$

$$\boxed{a_0 = \frac{1}{H_0}\frac{1}{|\Omega_0 - 1|^{1/2}}} \qquad (\Omega \neq 1)$$

(Matter dominated)

since we choose scale factor so that $k = \pm 1$, $a \sim$ "Hubble distance" if density is close to critical

③ Express Friedman solutions in terms of q, H

Recall $\quad a = \frac{1}{2} A \begin{bmatrix} 1 - \cos\eta \\ \cosh\eta - 1 \end{bmatrix}$

$$t = \frac{1}{2} A \begin{bmatrix} \eta - \sin\eta \\ \sinh\eta - \eta \end{bmatrix}$$

where $A = \frac{8\pi}{3}\rho a^3$ — which we can now express as

$$= 2H^2 q \left[\frac{1}{H|2q-1|^{1/2}}\right]^3 = \boxed{\frac{2q_0}{H_0 |2q_0 - 1|^{3/2}} = A}$$

(matter dominated)

④ Relate conformal time to redshift

$$(1+z)^{-1} = \frac{a}{a_0} = H_0 |2q - 1|^{\frac{1}{2}} \frac{q_0}{H_0 |2q_0 - 1|^{3/2}} \left(\qquad \right)$$

$$= \frac{q_0}{|2q_0 - 1|} \begin{pmatrix} 1 - \cos\eta \\ \cosh\eta - 1 \end{pmatrix}$$

or $$\boxed{(1+z)^{-1} = \frac{q_0}{2q_0 - 1}\left(1 - C(\eta)\right) \qquad C(\eta) = \begin{cases} \cos\eta \\ \cosh\eta \\ 1 \end{cases}}$$

or $$c(\eta) = -\frac{2q_0-1}{q_0(1+z)} + 1 = \boxed{\frac{q_0 z - q_0 + 1}{q_0(1+z)} = c(\eta)}$$

(5) Radial null geodesic:

Recall (p 39B)

$$r \equiv \Sigma(\chi) = \Sigma(\eta_0 - \eta_1) = \Sigma(\eta_0)\,C(\eta_1) - \Sigma(\eta_1)C(\eta_0)$$

$$\Sigma = \begin{cases} \sin\chi \\ \sinh\chi \\ \chi \end{cases}$$

Let's express radial coordinate of emission in terms of the redshift

For $K \neq 0$ $\quad \Sigma = |1 - c^2|^{1/2}$

$$1 - C = \frac{2q_0-1}{q_0(1+z)} \Rightarrow 1 + C = \frac{2q_0(1+z) - 2q_0 + 1}{q_0(1+z)}$$

$$= \frac{2q_0 z + 1}{q_0(1+z)}$$

$$\Sigma = \frac{|2q_0-1|^{1/2}(2q_0 z + 1)^{1/2}}{q_0(1+z)}$$

$$r = \Sigma(z=0)\,C(z) - \Sigma(z)\,C(z=0)$$

$$= \frac{|2q_0-1|^{1/2}}{q_0^2(1+z)}\left[q_0 z - q_0 + 1 - (1-q_0)(2q_0 z + 1)^{1/2}\right]$$

or $$\boxed{r = \frac{|2q_0-1|^{1/2}}{q_0^2(1+z)}\left[1 - q_0 + q_0 z - (1-q_0)(2q_0 z+1)^{1/2}\right]}$$

Note that we can expand [ ] in a power series as

$$[\quad] \simeq q_0^2 z + \tfrac{1}{2}(1-q_0)\, q_0^2 z^2 + \cdots$$

Hence

$$\boxed{r = \frac{|2q_0-1|^{1/2}}{(1+z)}\left[z + \tfrac{1}{2}(1-q_0)z^2 + \cdots\right]}$$

(We could also obtain this by expanding a in $\int \frac{dt}{a(t)} \cdots$)

(6) Age of the universe:

$$t = \tfrac{1}{2}A\begin{bmatrix} \eta - \sin\eta \\ \sinh\eta - \eta \end{bmatrix} \qquad \tfrac{1}{2}A = \frac{q_0}{H_0|2q_0-1|^{3/2}} = \frac{\Omega_0}{2H_0|\Omega_0-1|^{3/2}}$$

We saw above that $\Sigma_0 = \frac{2|\Omega_0-1|^{1/2}}{\Omega_0}$

Hence

$$t_0 = \frac{1}{H_0}\,\frac{\Omega_0}{2|\Omega_0-1|^{3/2}}\begin{bmatrix} -2\frac{(\Omega_0-1)^{1/2}}{\Omega_0} + \sin^{-1}\left(\frac{2(\Omega_0-1)^{1/2}}{\Omega_0}\right) \\ 2\frac{(1-\Omega_0)^{1/2}}{\Omega_0} - \sinh^{-1}\left(\frac{2(1-\Omega_0)^{1/2}}{\Omega_0}\right) \end{bmatrix}$$

and $t_0 = \frac{2}{3}\frac{1}{H_0}$ for $k=0$

Note: small $\Omega \Rightarrow t_0 \sim H_0^{-1}$

(universe is coasting, $H \sim$ constant)

E.g. open case:

$$t_0 = \frac{1}{H_0}\left[\frac{1}{1-\Omega_0} - \frac{\Omega_0}{2(1-\Omega_0)^{3/2}}\sinh^{-1}\left(\frac{2(1-\Omega_0)^{1/2}}{\Omega_0}\right)\right]$$

$$\Rightarrow \frac{2}{3}H_0^{-1} \text{ for } \Omega \to 1$$

$$\Rightarrow H_0^{-1}\left[1 - \frac{1}{2}\Omega_0 \sinh^{-1}(2/\Omega_0)\right]$$

$$\sinh^{-1} x \sim \ln 2x$$

$$\Rightarrow H_0^{-1}\left[1 - \frac{1}{2}\Omega_0 \ln(4/\Omega_0) + O(\Omega_0)\right]$$

Comoving Horizon

what angle is subtended now by the redshift z horizon volume?

Flat Space
$$r = \int \frac{dt}{a(t)} = \frac{1}{C} 3t^{\frac{1}{3}} \quad \text{if } a = Ct^{2/3} \text{ (mat dom)}$$

$$= \frac{3t}{a(t)} \Rightarrow d_{Hor} = 3t$$

$$\delta = \frac{r_{then}}{r_{now}} = \left(\frac{t}{t_0}\right)^{\frac{1}{3}} = (1+z)^{-\frac{1}{2}}$$

In general: Take $z \to \infty$ limit of r

$$\Rightarrow r = \frac{(\Omega-1)^{1/2}}{2\Omega}$$

How does Ω scale with z

$$\frac{8\pi}{3}\rho_{crit} = \frac{8\pi}{3}\rho - \frac{K}{a^2} \Rightarrow \frac{\Omega-1}{\Omega} = \frac{\rho-\rho_{crit}}{\rho} = \frac{3}{8\pi}\frac{K}{\rho a^2}$$

Mat dom $\Rightarrow$ $\frac{\Omega-1}{\Omega} \sim a$

We have $\delta = \dfrac{r_{then}}{r_{now}} = \left[\dfrac{(\Omega-1)/\Omega_{then}}{(\Omega-1)/\Omega_{now}}\right]^{\frac{1}{2}}\left[\dfrac{\Omega_{now}}{\Omega_{then}}\right]^{\frac{1}{2}}$

$$= (1+z)^{-\frac{1}{2}}\,\Omega_{now}^{\frac{1}{2}}$$

(since $\Omega_{then} \simeq 1$)

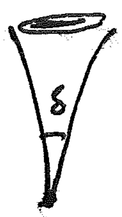
In the case of last scattering of μwave background — We know "T" ~ 4000° (relatively insensitive to Ω_b) ⇒ we know z

$\sqrt{1+z} \sim 40$

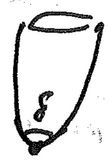
$\Rightarrow \delta \sim 1^\circ\,(\Omega_0^{\frac{1}{2}})$

Acoustic horizon is smaller by factor $\sim \frac{1}{3}$

Small Ω ⇒ open universe ⇒ smaller subtended angle



Large Ω ⇒ closed



⇒ larger angle

There are two effects of expansion on power radiated

Redshift $\Rightarrow$ Energy $\rightarrow$ Energy$/(1+z)$

And time:

$$\delta t_0 = \frac{a(t_0)}{a(t_1)}\,\delta t_1 = (1+z)\,\delta t_1$$

$\Rightarrow$ Flux $\rightarrow$ Flux$/(1+z)^2$

Sphere has radius

$$4\pi\,[a(t_0)\,r_1]^2$$

$$\Rightarrow \quad \text{Flux} = \frac{L}{4\pi r_1^2 a(t_0)^2}\,\frac{1}{(1+z)^2}$$

$$\Rightarrow \quad d_L = r_1\,a(t_0)(1+z) = r_1\,a(t_1)(1+z)^2$$

$$= d_A(1+z)^2$$

Magnitude - Redshift Relation

Suppose we have standard candles with luminosity L — How should the magnitude (apparent luminosity) be related to red shift z?

Relate d_L to red shift using

$$d_L = r_1\,a(t_0)(1+z)\,, \text{ where } a_0 = \frac{1}{H_0}\frac{1}{(2q_0-1)^{1/2}}$$

$$r_1 = \frac{(2q_0-1)^{1/2}}{q_0^{\,2}(1+z)}\left[1-q_0+q_0 z-(1-q_0)(2q_0 z+1)^{\frac{1}{2}}\right]$$

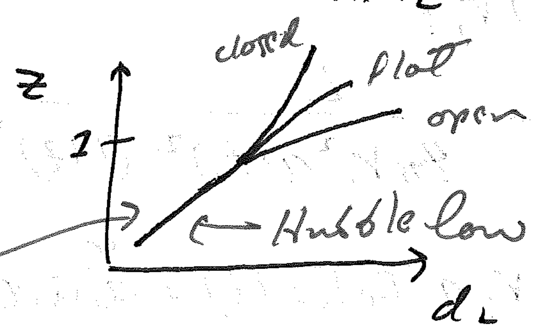
Hence

$$d_L = \frac{1}{H_0 q_0^2}\left[1 - q_0 + q_0 z - (1-q_0)(2q_0 z + 1)^{\frac{1}{2}}\right]$$

$$\boxed{d_L \simeq \frac{1}{H_0}\left[z + \tfrac{1}{2}(1-q_0)z^2 + \cdots\right]}$$

If we have standard candles:

$$\text{Flux} = \frac{L}{4\pi d_L^2} \quad \text{with } d_L \text{ as above}$$



$z \simeq H_0 d_L$

scatter here: peculiar velocity ~ 300 km/sec

For $z \gtrsim 1$ --

can discriminate different values of q_0

e.g. closed ⇒ high z sources appear to be "less distant"

geometrical focusing in closed universe, defocusing in open universe

But -- evolution

Current value of H_0? Need to measure distance to Virgo cluster

- Cepheids from Virgo with HST
- Supernovae

(Report z as $v = cz$)

Hubble: $H_0 \sim 500$ km/sec/Mpc

Baade, Sandage at Palomar, ---

$H_0 \sim 500$ Km/sec/Mpc $\simeq [2\times10^9 \text{yrs}]^{-1}$!? Universe younger than earth? "Age Problem"

Now $H_0 \sim 70$, $H_0^{-1} \simeq 14$ Gyears

For $\Omega = 1$ $\quad t_0 = \frac{2}{3}H_0^{-1} \simeq 9.5\,\text{Gyr}$ ⟵ cf Globular cluster ages 12-20 Gyr

$\Omega \ll 1$ $\quad t_0 = H_0^{-1}$

A mild age problem for the flat universe

What are limits on q_0? I don't have up to date number -- crudely $q_0 \doteq 1$, $\Omega_0 \doteq 2$

What about Λ?

$$8\pi T_{\mu\nu} \to 8\pi T_{\mu\nu} - \Lambda g_{\mu\nu}$$

i.e. $\rho \to \rho + \rho_{vac}$

$$\rho_{vac} = \frac{\Lambda}{8\pi}$$

$$\frac{\dot a^2}{a^2} = \frac{8\pi}{3}(\rho + \rho_{vac}) - \frac{K}{a^2}$$

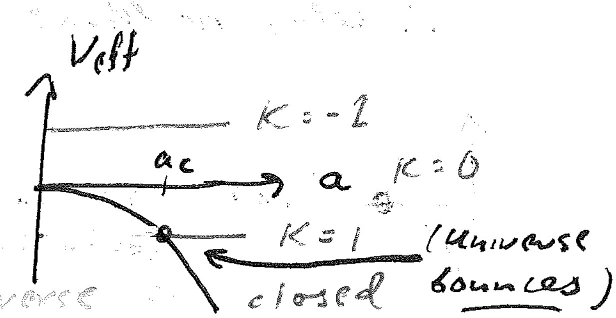
ρ: diluted by expansion; ρ_{vac}: constant

$$P_{vac} = -\frac{\Lambda}{8\pi} = -\rho_{vac}$$

For example, suppose $\rho_{matter} = 0$

$$\Rightarrow \dot a^2 = \left(\frac{8\pi}{3}\rho_{vac}\right)a^2 - K$$

The effective potential is $V_{eff} = -a^2/a_c^2$



Einstein's (unstable) static universe

(NOTE: closed universe can expand forever)

the $K=0$ case is the DeSitter universe

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}\rho_{vac} = H^2 \qquad \Rightarrow \qquad \frac{\dot a}{a} = \pm H$$

i.e. $a = e^{Ht}$, e^{-Ht}

For $\Lambda > 0$

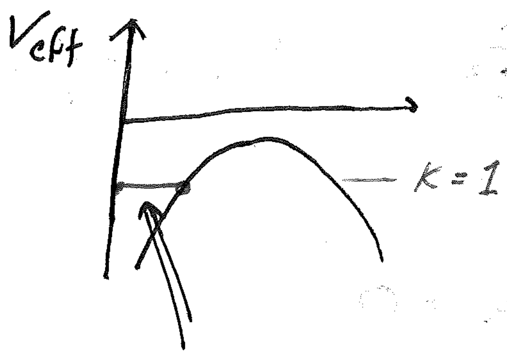
ρ_{vac} has a repulsive effect $\Rightarrow$ exponential "inflation"

(comoving galaxies are blown apart)

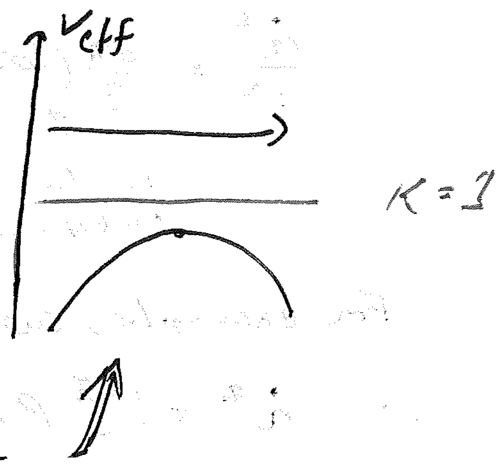
What about nonrelativistic (pressureless) matter and Λ

$$\Rightarrow \quad \frac{1}{2}\dot a^2 - \frac{1}{2}\frac{A}{a} - \frac{1}{2}H_{vac}^2 a^2 = -\frac{1}{2}K$$

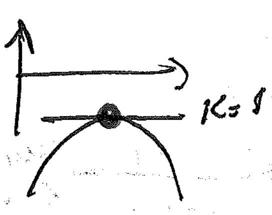
i.e. $V_{eff} = -\frac{1}{2}H_{vac}^2 a^2 - \frac{1}{2}\frac{A}{a}$] Has a max for $a^3 = \frac{A}{2H^2}$



Λ delays recollapse



[Expansion slows and then speeds up again!



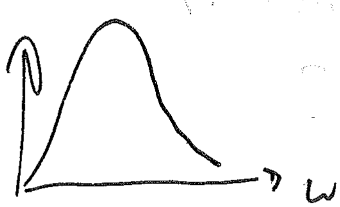
of course, this also happens in the flat case

The Cosmic Microwave Background Radiation

(CMBR, CBR)

Great (1965) discovery!
Universe has a temperature

Filled with photons at $\lambda \sim 0.1-3$ cm



COBE (Cosmic Background Explorer)
$\Rightarrow$ a beautiful fit to Black Body curve (1990)

We know $T \sim 2.73 \pm .01\ ^\circ K$

And we know temperature at earlier epoch

Expansion is adiabatic --- gets modified at high T because more species share entropy

$$T_{then} : \frac{a_{now}}{a_{then}} T_{now} \quad \text{(Redshift)}$$

$$\rho_{\gamma_0} = \frac{\pi^2}{15}(\hbar c)^{-3}(KT)^4 = 4.7 \times 10^{-34}\ g/cm^3$$

- Today -- universe is matter dominated
- But there are many more γ's than p, n, e's today

How to be quantitative?

Synthesis of Light Elements

^2H ^3He ^4He ^7Li cooked in early universe (heavier elements produced in stars)

Scenario:

At $T \simeq 1\,MeV$ n, p abundance in equilibrium

Boltzman $X_n = \frac{n}{n+p} = \frac{1}{1+e^{Q/T}}$ $Q = 1.29\,MeV$

E.g. $\bar{\nu} p \leftrightarrow e^+ n$ weak interactions
$\nu n \leftrightarrow e^- p$ convert $n \leftrightarrow p$

But -- key freeze out -- when?

Here universe is $\left\{ \begin{array}{l} \text{radiation dominated} \\ \Omega \sim 1 \end{array} \right\}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi}{3} G \rho_{rad} = \frac{8\pi}{3} G (\sigma g_* T^4)$$

$\sigma = \frac{\pi^2}{15} (\hbar c)^{-3} k^4$

g_*: effective # of species ($7/8$ for Fermi x 2 spin states)
— ν's contribute also

$$\frac{dT}{dt} = -\sqrt{\frac{8\pi}{3} G \sigma g_*}\, T^3$$

$$\Rightarrow \frac{1}{2T^2} = \sqrt{8\pi/3\, G \sigma g_*}\; t$$

$$\text{or}\quad t = \frac{1}{\sqrt{\frac{32\pi}{3} G \sigma g_* T^4}} = \frac{1}{\sqrt{\frac{32\pi}{3} G \rho_{rad}}}$$

Relates age of the universe to its temperature

Need to study: reaction kinetics in expanding universe —

but let's do an order of magnitude estimate of weak interaction freeze out

Equate expansion rate and reaction rate

$$\underbrace{G_F^2 T^2}_{(\sigma v)_{weak}} \underbrace{T^3}_{n_{\nu,e}} \sim \underbrace{T^2/m_p}_{\text{Expansion rate} \sim \dot{a}/a} \sim \sqrt{G\rho} \qquad m_p \sim \frac{1}{\sqrt{G}} \sim 10^{19}\,\text{GeV}$$

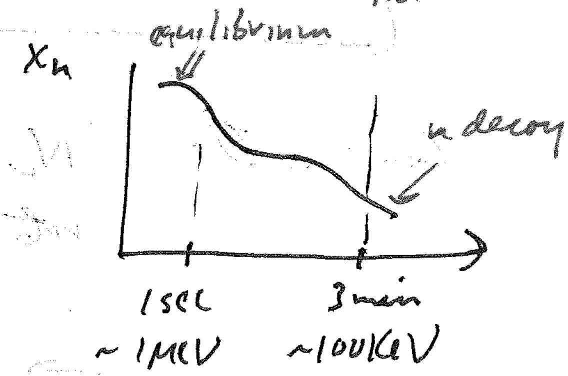
$$\Rightarrow T^3 \sim \frac{M_{weak}^4}{m_p} \quad \text{or} \quad T \sim \underbrace{M_{weak}}_{\sim 300\,\text{GeV}} \underbrace{\left(\frac{M_{weak}}{m_p}\right)^{1/3}}_{\sim 3\times10^{-6}} \sim 1\,\text{MeV}$$

coincidentally $T_{f.o.} \sim \Delta$

Now — neutron decay

$$X_n = \frac{n}{n+p} = e^{-t/t_N} X_{n,0}$$

$$t_N = 1013\,\text{sec}$$



Meanwhile — nuclear reactions

$n\,p \leftrightarrow D\,\gamma$ Bottleneck $\Longleftarrow$ since D is weakly bound $B_D \sim 2.2\,\text{MeV}$, equilibrium D abundance is low

$\cancel{n\,D \leftrightarrow {}^3H\,\gamma}$

$\cancel{p\,D \leftrightarrow {}^3He\,\gamma}$

$\cancel{n\,{}^3He \leftrightarrow {}^4He}$

$D\,D \leftrightarrow {}^3He\,n$

$\ {}^3H\,p$

${}^3H\,D \leftrightarrow {}^4He\,n$

${}^3He\,D \leftrightarrow {}^4He\,p$

Then until nucleosynthesis can proceed at $T \sim 100\,\text{KeV}$

$Y = \quad X_{He^4} = 2\, X_n$ (just before nucleosynthesis)

$\Uparrow$

More baryons $\Rightarrow$ higher T

$\Longleftarrow \qquad \Rightarrow$ More He

But ^3He

^2H

^7Li

much more sensitive $\Big] \Rightarrow$ compare to primeval abundance

(e.g. $Y_{He^4} \sim .22$)

$$\boxed{\Omega_B h^2 \simeq 0.013 \pm 0.003}$$

Comment: N_ν also contributes to expansion rate (more ν's $\Rightarrow$ faster expansion $\Rightarrow$ more ^4He

$\Rightarrow N_\nu \sim 3$ (confirmed by LEP)

4 abundances all fit

$\Rightarrow$ consistency check!

And: Today $\dfrac{\rho_B}{\rho_\gamma} \simeq 600$

$$\rho_B = \Omega_B \rho_c = \Omega_B \frac{3 H_0^2}{8\pi G} = 1.88 (\Omega_B h^2) \times 10^{-29}\, g/cm^3$$

Can express as

$$\eta = \frac{n_N}{n_\gamma} \sim 3.5 \pm 0.8 \times 10^{-10}$$

Preserved by adiabatic expansion —

Or express as entropy per baryon $\sim 10^{10}$

Big bang was hot

Equivalently entropy [illegible] preserved by adiabatic expansion

$\boxed{\text{entropy per photon} \sim 3.6}$

Note:

For $T \gg 1$ GeV

$n_B \sim n_{\bar{B}} \sim n_\gamma$

so $\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} \sim \text{few} \times 10^{-10}$

A tiny baryon excess

$$\boxed{\text{Baryon Excess: Why } \frac{n_n}{n_\gamma} \sim 3 \times 10^{-10} \text{ ??}}$$

Dark Matter

Is there other stuff besides ordinary matter??

E.g. we can weigh rich clusters of galaxies by measuring velocity dispersion --

$$\frac{\text{Mass}}{\text{Light}} \sim 300 \pm 100\, h \frac{M_\odot}{L_\odot}$$

$$\Rightarrow \boxed{\Omega \sim 0.1 - 0.2}$$

stuff that clusters on scale $\lesssim 10 h^{-1}$ Mp

Probably can't be all baryons
Even $h = \frac{1}{2} \Rightarrow \Omega_B < 4 \times .016 = .064$

$\boxed{\text{What is the Dark Matter??}}$

seems to make up halos of <u>galaxies</u> as well ...

<u>Not</u> ordinary stuff

Anisotropy of the CBR

First — there is the dipole: it tells us where we are going!

$T(\theta) = T_0(1 + v\cos\theta)$

(the "Aether Drift" finally detected —)

Correct for solar motion relative to local group

$\Rightarrow \quad v = 600 \ \frac{\text{Km}}{\text{sec}}$] Large compared to local scatter $\Rightarrow$ coherent motion

What are we falling toward?

Virgo? ($cz \sim 1000$ Km/sec)

Resultant of Virgo and "Great Attractor"?
at $cz \sim 5000$ Km/sec

sky ~ factor ~ 10^{-3} hotter in forward direction

<u>Quadrupole:</u> $\underline{\frac{\Delta T}{T} \sim 5 \times 10^{-6}}$ (COBE 1992)

<u>Very</u> <u>isotropic</u>. <u>Remarkable!</u>

Last Scattering Surface:

The microwave photons have been traveling a long time! (universe is transparent)

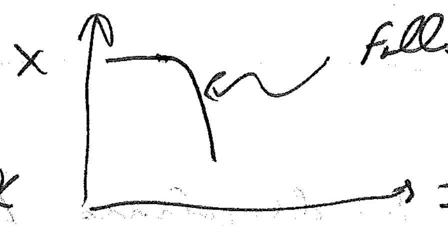
When H was ionized, γ scattering was frequent

Saha Eqn: equilibrium for $e\, p \leftrightarrow H\, \gamma$

Binding energy

$$\frac{n_e n_p}{n_H(n_H+n_p)} = \frac{x^2}{1-x} = \frac{(2\pi m_e kT)^{3/2}}{(n_H+n_p)(2\pi\hbar)^3} e^{-B/kT}$$

$x = \frac{n_e}{n_{total}} = \frac{n_p}{n_{total}}$



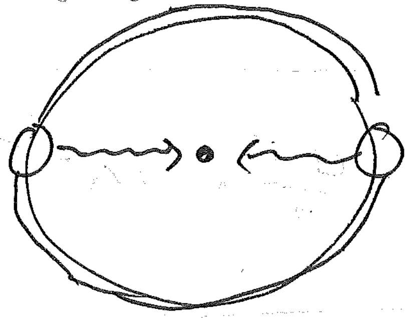
At $x = \frac{1}{2}$:

$\Omega_B h^2 = .013 \Rightarrow T = 3700°K$, $z = 1360$

$\Omega_B h^2 \cong 1 \Rightarrow T \sim 4000°K$, $z \sim 1520$

So not so sensitive to Ω_B (because of exponential)

(This is inexplicably called "recombination")



Consider photons reaching us today from Last scattering ($z \sim 1400$) surface

Recall homework (or p. 40C): Matter dominated $\Rightarrow$

$$\frac{r_{H,then}}{r_{H,now}} \simeq (1+z)^{-\frac{1}{2}}\,\Omega_{now}^{\frac{1}{2}} \sim \frac{1}{40}(.2)^{\frac{1}{2}} \sim .01$$

coordinate size of horizon

These γ's are communicating for the first time now

Were ~ 100 horizon lengths apart at time of emission —

$a \sim t^{2/3} \sim r_H^{2/3}$

Light travels faster than expansion

How did they know to have same temp. to $\gtrsim 10^{-5}$ accuracy!!

Horizon Problem:

$\boxed{\text{Why is the universe so homogeneous and isotropic?}}$

But the deviations from perfect isotropy are also of fundamental significance

(These become fluctuations in density that can grow into galaxies and other structures

Anisotropy:

$\boxed{\text{What is the origin of } \frac{\Delta T}{T} \sim 10^{-5}\text{?}}$

E.g. $\left.\frac{\delta \rho}{\rho}\right|_{\text{Horizon}} \sim 10^{-5}$

Recall — we also saw (HW + page 40C)

$$\frac{8\pi}{3}\rho_{cr} = \frac{8\pi}{3}\rho - \frac{k}{a^2} \Rightarrow \frac{\Omega-1}{\Omega} = \frac{\rho-\rho_{crit}}{\rho} = \frac{3}{8\pi}\frac{k}{\rho a^2}$$

Hence $\frac{\Omega-1}{\Omega} \sim$ a/a_0 matter dom, a^2/a_0^2 rad dom

Mysterious that $\Omega \sim 0.1-1$ today, because $\Omega = 1$ is unstable -- either
- recollapsed long ago
- $\Omega \lll 1$ today

E.g. at $T \sim 1\,\mathrm{MeV}$, $t \sim 1\,\mathrm{sec}$

$$\frac{\Omega-1}{\Omega} \sim \frac{1}{1+z_{eq}}\left(\frac{T_{eq}}{1\,\mathrm{MeV}}\right)^2$$

$$|\Omega - 1| \lesssim 10^{-16}\ !$$

How did the universe tune Ω so well?

$$\boxed{\text{Flatness: why } |\Omega-1| \lll 1 \text{ in the early universe}}$$

Answers —

In brief

- $\Delta B \neq 0$ and CP violation
- DM? Could it be LSP?
- Horizon + flatness: Because universe expanded superluminally in its early history (inflation)

- And inflation stretched quantum fluctuations of fields to cosmological size!

we might add the puzzle:

Cosmological Constant:
Why $\rho_{vac}/m_{pl}^4 \lesssim 10^{-122}$?

(and why $\rho_{vac} \neq 0$??!)

Observations of μ wave anisotropy

E.g. MAP satellite will map the μ wave sky with ~ 10 minute angular resolution (also various ground based exps — balloon)

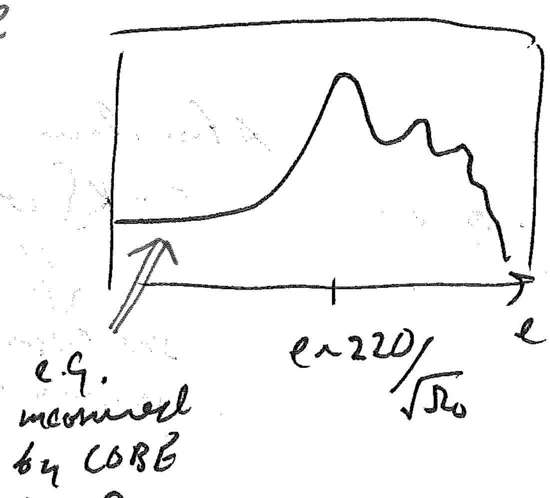
$$\frac{\Delta T}{T} = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

$$\left\langle \frac{\Delta T}{T}(\theta,\phi)\, \frac{\Delta T}{T}(0,0) \right\rangle = \frac{1}{4\pi} \sum_\ell (2\ell+1) \underbrace{\langle (a_{\ell m})^2 \rangle}_{C_\ell} P_\ell(\cos\theta)$$

So -- measure $\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle_\ell^{\frac{1}{2}} \Leftarrow$ proper angular scale $\theta \sim \pi/\ell$

Plot $\ell(\ell+1)C_\ell$ = power spectrum (i.e. flat if fluctuations are "scale free")

Expect to see



what is origin of this structure?

The structure arises at the angular scale of the comoving horizon (at decoupling), which we computed to be --

$$\theta \simeq (1+z_*)^{-\frac{1}{2}}\,\Omega_0^{1/2} \sim .027\,\Omega_0^{\frac{1}{2}}$$

(for $z_* \sim 1400$)

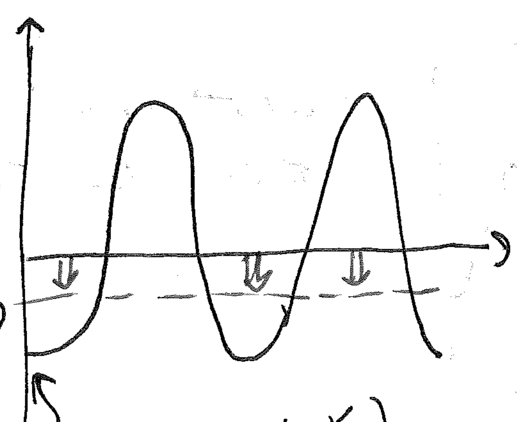
$$\Rightarrow \ell \sim \pi/\theta \sim 120\,\Omega_0^{-\frac{1}{2}}$$

– last scatt is a little later ~ $z \sim 1100$

why not 220 instead -- e.g. sound speed < c

The point is that the fluctuations in energy density undergo acoustic oscillations inside the "sound horizon" (where $v_s \sim \frac{1}{\sqrt{3}} c$ for radiation fluid)

The oscillations stop abruptly at decoupling (recombination)



A particular normal mode with wave number K

mass of baryons pulls down equilibrium

Initial (stochastic) fluctuation

A function of $s = K\int v_s\,d\eta$

conformal time (because then behaves like flat space wave)

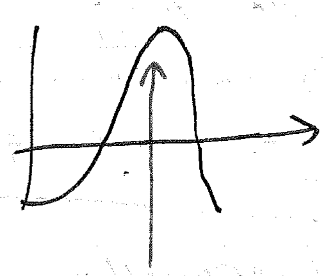
speed of sound may vary

Both compression and rarefaction are seen as a peak in $\left\langle \left(\frac{\Delta T}{T}\right)^2 \right\rangle^{\frac{1}{2}}$

↳ odd peaks are bigger than even peaks

The observed $\frac{\Delta T}{T}$ is a combination of:

- Acoustic oscillation of T
- Doppler effect, since last scattering surface is moving (so called "Doppler peak" — but that is really a <u>misnomer</u> — since other effects are really more important — Doppler is out of phase with dominant effect)
- Grav. red shift (Sachs-Wolfe) because photons climb out of grav potential well
- Photon diffusion — which suppresses acoustic oscillations for large l

$\frac{\Delta T}{T}$ 

First peak occurs where
$s = k\int c_s d\eta = \pi$

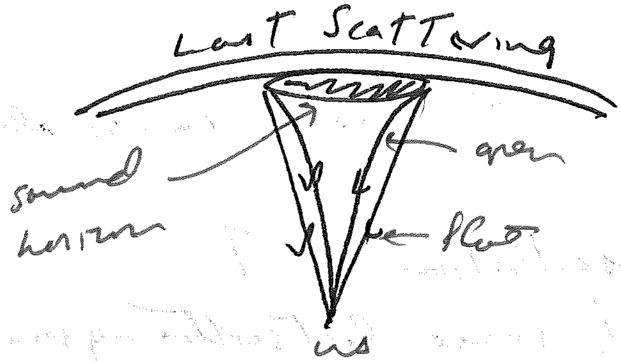
And — various secondary effects —
- integrated SW
- rescattering

E.g. $\frac{2\pi}{\lambda}\frac{1}{\sqrt{3}}\eta = \pi \Rightarrow \lambda = \frac{2}{\sqrt{3}}\eta$

and recall $\eta = \frac{2|\Omega-1|^{\frac{1}{2}}}{\Omega}$ (page 40B)

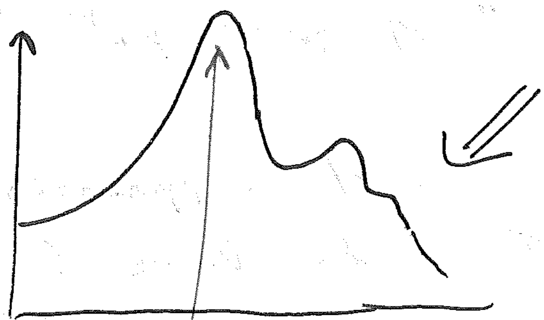
$= 4r_H \Rightarrow \lambda \simeq \frac{8}{3} r_H$ (sound wavelength)

Anyway — comoving scale of first peak ~ sound horizon at last scattering



$\theta_{peak} \sim \sqrt{\Omega_0}$

This is a geometrical effect -- the angular scale of comoving horizon is smaller in an open universe —



These peaks are suppressed by photon diffusion (Silk damping)

i.e. γ's can travel further than a wavelength —

Peak height $\sim \Omega_B$

Why? — determines inertia of the oscillating stuff, and how far it is displaced from naive equilibrium — i.e. how far does it collapse before pressure makes it rebound

WARNING: "Cosmic Confusion"

Note another coincidence

$\rho_B \sim \rho_\gamma$ at $z \sim z_*$

Spacing of peaks is sensitive to h.. why? Is it — how c_s varies close to last scatt —

Inflation!

What if, in an earlier epoch -- vacuum energy dominated

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G \rho_{vac} = \text{const} = H^2$$

= "Hubble constant" really is constant

$$\Rightarrow \quad a(t) = a_0 e^{H(t-t_0)}$$

exponential expansion

Coordinate distance traveled by photon

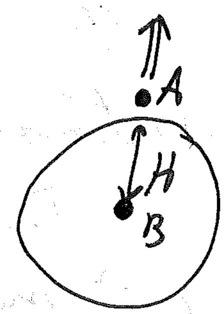
$$r = \int_{t_i}^{t_f} \frac{dt}{a(t)} = \frac{1}{a_0}\int_{t_i}^{t_f} dt\, e^{-H(t-t_0)}$$

$$= \frac{1}{a_0 H}\left[e^{-H(t_i-t_0)} - e^{-H(t_f-t_0)}\right]$$

$$\rightarrow \frac{1}{a_0 H} e^{-H(t_i-t_0)} = \frac{1}{a_i H}$$

Hence $\underbrace{a_i r}_{} = \frac{1}{H}$

This is physical horizon size at time of emission



If proper distance between A and B > H -- signal will never catch up

= Expansion is superluminal

In Friedman universe — the horizon catches up with expansion —

But in inflationary universe, the expansion overtakes the horizon

Comoving $L(t) = L(0)\, e^{Ht}$

[comoving observers are pulled exponentially far apart — even though once in causal contact

solves horizon problem

what about flatness?

$$\frac{\Omega - 1}{\Omega} = \frac{3k}{8\pi G \rho a^2}$$

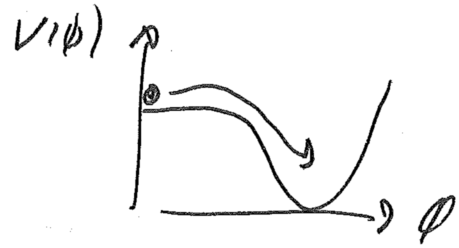
In Friedman universe ρa^2 was decreasing, so Ω fell away from 1

But in inflationary universe $\Omega \to 1.0000$ — because ρa^2 increases exponentially —

[Aside: Energy conservation?
Total energy = Matter + Grav Pot. Energy = 0
Universe is a free lunch (blown apart by negative pressure)

But -- we require a graceful exit

$\rho_{vac} \rightarrow \rho_{rad}$



Must generate

$n_B/n_\gamma \sim 3\times10^{-10}$ after

universe "reheats"

E.g. slow rolling of an "inflaton" field

(any initial n_B has been "inflated away")

A universe can be created "from nothing," because it carries no conserved quantities —

Bonus

- Removes unwanted relics (magnetic monopoles)
- Generates $\Delta T/T$!

  Q. fluctuations blown to enormous size

  – $(\delta\rho/\rho)_{Horizon}$ = constant. (scale free) (each scale behaves the same way as it leaves horizon)

  – Gaussian statistics?

- Universe could be eternal (always inflating somewhere!)
- Predict $\Omega_0 + \Omega_\Lambda = 1$ — Universe should be flat