and has a coupling to the Higgs boson

$$-\frac{m_e}{v}\phi'\bar{e}e.$$

Since $m_e = .511\,\mathrm{MeV}$ $v = 247\,\mathrm{GeV}$, we see

$$f_e = \sqrt{2}\,\frac{m_e}{v} = 2.9\times 10^{-6}$$

There is no explanation within the model for why this dimensionless parameter is so small.

Note that, in the standard model, there is no right handed neutrino (by assumption) and so the neutrino is exactly massless. (That is, any neutrino mass must be attributed to new physics not described by the model)

More Generations

We will now incorporate the muon and its neutrino into the model. Since the eak interactions of μ and ν_μ are similar to those of e and ν_e, we hypothesize that they transform the same way under $SU(2)_L\times U(1)_Y$:

$$\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \qquad \mu_R$$
$$Y=-\tfrac{1}{2} \qquad Y=-1$$

It is automatic that the charged and neutral weak currents have the same form and strength for e's and μ's. (this is called e-μ universality) E.g., W^+ couples as

$$\frac{ig}{\sqrt{2}} W_\mu^+ \left(\bar{\nu}_{eL} \gamma^\mu e_L + \bar{\nu}_{\mu L} \gamma^\mu \mu_L \right)$$

Of course, this universality will extend if we add still more leptons transforming the same way (τ, ν_τ, etc.)

Now we can write down more independent Yukawa couplings. In unitary gauge we have

$$\mathcal{L}_{Yuk} = -\frac{1}{\sqrt{2}} \Big[f_{ee} \bar{e}_L e_R + f_{e\mu} \bar{e}_L \mu_R + f_{\mu e} \bar{\mu}_L e_R + f_{\mu\mu} \bar{\mu}_L \mu_R \Big] (v + \phi')$$

$$+ h.c.$$

(there are four independent couplings)
If we have N identical lepton generations, then

$$\mathcal{L}_{Yuk} = -\frac{1}{\sqrt{2}} \left(f_{\alpha\beta} \cdot \bar{l}_{L\alpha} l_{R\beta} \right) (v + \phi') + h.c.$$

— there are N^2 Yukawa couplings.
But many of these apparent free parameters are illusory; they can be absorbed into redefinitions of the fields.

We may appeal to a theorem of linear algebra: If f is an $N \times N$ matrix, then there are unitary V_L, V_R such that

$$f' = V_L^{+} f V_R = \text{diagonal, real, and nonnegative}$$

We may change variables as

$$l_{R\alpha} = V_{R\alpha\beta} l'_{R\beta}$$
$$l_{L\alpha} = V_{L\alpha\beta} l'_{L\beta}$$

This leaves invariant the kinetic terms

$$\bar{l}_{R\alpha} i \not{\partial} l_{R\alpha} + \bar{l}_{L\alpha} i \not{\partial} l_{L\alpha}$$
$$= \bar{l}'_{R\alpha} i \not{\partial} l'_{R\alpha} + \bar{l}'_{L\alpha} i \not{\partial} l'_{L\alpha},$$

but transforms the Yukawa coupling to the diagonal form

$$\mathcal{L}_{Yuk} = \sum_{\alpha} m_\alpha \bar{l}_\alpha l_\alpha \left(1 + \frac{\phi'}{v}\right)$$

So the actual number of free parameters is N -- the N charged lepton masses. And notice that when we diagonalize the lepton mass matrix, we also diagonalize the Yukawa couplings of ϕ' -- that is, the Higgs boson coupling preserves each of the N varieties of <u>lepton number</u>.

Now, the form of the weak currents

$$\sum_\alpha (\bar{\nu}_{L\alpha} \; \bar{l}_{L\alpha}) (ig W_\mu^a T^a + ig' B_\mu Y)\gamma^\mu \begin{pmatrix} \nu_{L\alpha} \\ l_{L\alpha} \end{pmatrix}$$

may appear to be messed up by our redefinition of the $l_{L\alpha}$'s, but it too is unchanged in form if we simultaneously rotate the neutrinos,

$$\nu_{L\alpha} = V_{L\alpha\beta} \nu'_{L\beta}.$$

In other words, we decide "which charged lepton is which" by diagonalizing the lepton mass matrix. Then we identify e.g. the "electron neutrino" as that neutrino which is in the same weak doublet with e_L. With these definitions, the standard model conserves e number, μ number, τ number, etc.

Decay of W and Z

Even in this fictitious leptonic world, the W and Z are not stable. They can decay <u>leptonically</u>: E.g.

$$W^- \longrightarrow e^- \bar{\nu}_e$$

$$Z^0 \longrightarrow e^+ e^-$$

$$\bar{\nu}_e \nu_e$$

(You will calculate rates for a homework problem) These leptonic decay modes are what makes W's and Z's identifiable in hadronic collider experiments.

The Higgs Boson

There is just one physical scalar, the ϕ', that avoids being eaten by a gauge boson. In unitary gauge,

$$D^\mu\phi^\dagger D_\mu\phi$$

$$= \frac{1}{2}\left|\left[\partial_\mu + \frac{ig}{\sqrt{2}}(W_\mu^+T^+ + W_\mu^-T^-) + ieA_\mu Q + \frac{ie}{\sin\theta\cos\theta}Z_\mu(T^3 - \sin^2\theta Q)\right]\begin{pmatrix} v+\phi' \\ 0 \end{pmatrix}\right|^2$$

$$= \frac{1}{2}\left|\begin{pmatrix} \partial_\mu\phi' \\ 0 \end{pmatrix} + \frac{ig}{\sqrt{2}}W_\mu^-\begin{pmatrix} 0 \\ v+\phi' \end{pmatrix} + \frac{ieZ_\mu}{2\sin\theta\cos\theta}\begin{pmatrix} v+\phi' \\ 0 \end{pmatrix}\right|^2$$

$$= \frac{1}{4}g^2 W_\mu^- W^{\mu+}(v+\phi')^2 + \frac{e^2}{8\sin^2\theta\cos^2\theta}Z_\mu Z^\mu(v+\phi')^2$$

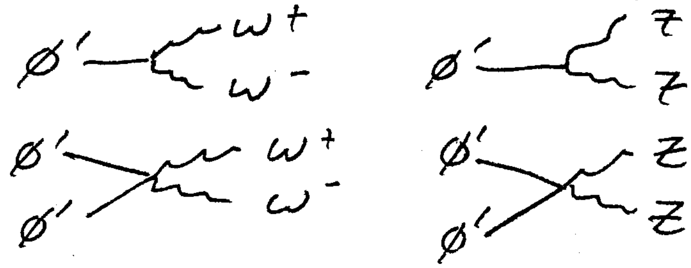
$$+ \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

-- (there is no bilinear coupling of ϕ' to Z because ϕ' is <u>real</u>)

$$= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \mu_W^2 W_\mu^- W^{\mu+}\left(1+\frac{\phi'}{v}\right)^2$$

$$+ \frac{1}{2}\mu_Z^2 Z_\mu Z^\mu(1+\phi'/v)^2$$

So the Higgs boson has couplings



The Higgs will decay to W or Z pairs if it is heavy enough.

How heavy is it? In unitary gauge

$$-\frac{\lambda}{2}\left(\phi^*\phi - \frac{v^2}{2}\right)^2 = -\frac{\lambda}{8}\left(\phi'^2 + 2\phi' v\right)^2$$
$$= -\frac{1}{2}(\lambda v^2)\phi'^2 - \frac{1}{2}\lambda v \phi'^3 - \frac{\lambda}{8}\phi'^4$$

So $$m_H^2 = \lambda v^2$$

Though v is known, λ, the scalar self-coupling, is a free parameter, so there is no prediction in the standard model for the Higgs mass.

As we have already seen the ϕ' also has Yukawa couplings to the charged leptons

$$-\frac{\phi'}{v}\left(m_e \bar{e}e + m_\mu \bar{\mu}\mu + m_\tau \bar{\tau}\tau\right)$$

-- the heavier the lepton the stronger the coupling. Thus the decay

$$\phi' \to \tau^+\tau^-$$

has a much higher rate than $\phi' \to \mu^+\mu^-, e^+e^-$ -- which are suppressed by $(m_\mu/m_\tau)^2$, $(m_e/m_\tau)^2$ respectively.

How to Include Quarks

Recall (e.g., page 6.44) that the hadronic weak current has the structure

$$J^\mu_{Hadron} = \bar{u}_L \gamma^\mu d_L + \text{more generations}$$

(up to Cabibbo mixing -- more about that a little later). This structure suggests that u_L, d_L, like ν_L, e_L form a weak $SU(2)_L$ doublet

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}.$$

The electric charges of the quarks are

$$Q = T_3 + Y = \begin{pmatrix} 2/3 & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}+Y & 0 \\ 0 & -\frac{1}{2}+Y \end{pmatrix},$$

so we see that the quark doublet should be assigned the weak hypercharge

$$Y = \frac{1}{6}$$

Once we decide that the LH quarks form an $SU(2)_L$ doublet, we have accounted for the origin of lepton-hadron universality. That is, the charge-current weak interactions of leptons and quarks have the same form and strength, since the $W^{\pm}$ couple to the $SU(2)_L$ generators $T^{\pm}$.

The neutral current weak interactions do not exhibit lepton-hadron universality, since the Z^0 couples to the $SU(2)_L \times U(1)_Y$ generator

$$\frac{e}{\sin\theta\cos\theta}\left(T^3 - Q\sin^2\theta\right)$$

-- which has a different strength for e, ν, u, d, since all have different electric charges.

Quark Masses

In addition to the LH quarks, there are of course RH quarks -- the Y assignments of the quarks are

$$q_L^{\frac{1}{6}} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}^{\frac{1}{6}} \qquad \underbrace{u_R^{2/3} \quad d_R^{-\frac{1}{3}}}_{SU(2)_L \text{ singlets}} .$$

What Yukawa couplings of the quarks to the Higgs doublet --

$$\phi^{-\frac{1}{2}}, \quad (\phi^c)^{\frac{1}{2}}$$

are allowed by the gauge symmetry? There are --

$$\mathcal{L}_{Yuk} = -f_u \bar{q}_L \phi u_R - f_d \bar{q}_L \phi^c d_R + h.c.$$

(and note that Yukawa couplings of quarks to leptons are forbidden). In unitary gauge

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi' + v \\ 0 \end{pmatrix},$$

this becomes

$$\mathcal{L}_{Yuk} = \frac{-1}{\sqrt{2}}(\phi' + v)\left(f_u \bar{u}u + f_d \bar{d}d \right)$$

$$= -m_u \bar{u}u - m_d \bar{d}d$$

$$- \frac{m_u}{v} \bar{u}u\phi' - \frac{m_d}{v} \bar{d}d\phi',$$

where

$$m_u = \frac{1}{\sqrt{2}} f_u v$$

$$m_d = \frac{1}{\sqrt{2}} f_d d$$

Both quarks acquire mass, and have couplings to the the Higgs boson ϕ' proportional to their mass.

Two Quark Generations --
Cabibbo Universality and the GIM Mechanism

As with leptons, the $SU(2)_L \times U(1)_Y$ multiplet structure of the quarks repeats itself:

We have $\begin{pmatrix} c_L \\ s_L \end{pmatrix}^{\frac{1}{6}}$ $\quad c_R^{2/3} \quad s_R^{-\frac{1}{3}}$

Let's denote the quarks by

$$q_{1L}, q_{2L}, \quad u_{1R}, u_{2R}, \quad d_{1R}\ d_{2R}$$

-- the index 1, 2 labeling the generation.

Now there are two 2×2 matrices of allowed Yukawa couplings:

$$\mathcal{L}_{Yuk} = -\bar{Q}_L \phi F_U U_R - \bar{Q}_L \phi^c F_D D_R + h.c.$$

(Here Q_L, U_R, D_R carry generation indices 1, 2, and $F_{U,D}$ are 2×2 matrices) In unitary gauge this becomes

$$\mathcal{L}_{Yuk} = -\left(1+\frac{\phi'}{v}\right)\left[\bar{U}_L M_U U_R + \bar{D}_L M_D D_R + h.c.\right]$$

Now, the mass matrices $M_{U,D}$ (and the Yukawa couplings) can be diagonalized by performing independent unitary changes of basis for $U_{L,R}$, $D_{L,R}$. That is, we write

$$U_L = V_{LU} U_L'$$

$$U_R = V_{RU} U_R'$$

$$D_L = V_{LD} D_L'$$

$$D_R = V_{RD} D_R'$$

so that the new mass matrices are

$$\mathcal{M}_U' = V_{LU}^{+} \mathcal{M}_U V_{RU} = \text{diagonal, real, nonneg.}$$
$$\mathcal{M}_D' = V_{LD}^{+} \mathcal{M}_D V_{RD} = \text{diagonal, real, nonneg.}$$

This change of basis leaves the kinetic terms for the quarks unchanged, but what happens to the weak currents?

Charged Currents:

In the original basis, the $SU(2)_L$ doublets were

$$\begin{pmatrix} u_{1L} \\ d_{1L} \end{pmatrix}, \quad \begin{pmatrix} u_{2L} \\ d_{2L} \end{pmatrix} \quad = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$$

(generation index understood)

In terms of the mass eigenstates, this becomes

$$\begin{pmatrix} V_{LU}\, U_L' \\ V_{LD}\, D_L' \end{pmatrix},$$

or, by taking appropriate linear combinations

$$\begin{pmatrix} U_L \\ V D_L \end{pmatrix} \quad \text{where } V = V_{LU}^{+} V_{LD}$$

(and I have dropped primes -- unprimed U, D now denote mass eigenstates).

We recall now that $W_\mu^{\pm}$ have couplings

$$\frac{g}{\sqrt{2}}\left(\gamma^\mu W_\mu^+ T^+ + \gamma^\mu W_\mu^- T^-\right),$$

so e.g. W_μ^+ couples to the current

$$\frac{g}{\sqrt{2}}\, \bar{U}_L \gamma^\mu V D_L$$

where V is a 2×2 unitary matrix

By making appropriate phase choices for the quark fields in

$$(\bar{u}\ \bar{c})\begin{pmatrix} e^{i(\gamma+\alpha)}\cos\theta & + e^{i\beta}\sin\theta \\ -e^{-i\beta}\sin\theta & e^{i(\gamma-\alpha)}\cos\theta\end{pmatrix}\begin{pmatrix} d \\ s\end{pmatrix}$$

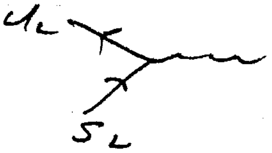
we can remove phases from the matrix V, and hence obtain:

$$\frac{g}{\sqrt{2}}\, W_\mu^+\big(\cos\theta\ \bar{u}_L\gamma^\mu d_L + \sin\theta\ \bar{u}_L\gamma^\mu s_L$$
$$- \sin\theta\ \bar{c}_L\gamma^\mu d_L + \cos\theta\ \bar{c}_L\gamma^\mu s_L\big)$$

The parameter θ is called the <u>Cabibbo angle</u> (often denoted θ_c to distinguish it from $\theta_W = \arctan(g'/g)$)

We see that, with two generations, lepton-hadron universality becomes "Cabibbo universality." The rates for <u>strangeness - preserving</u> hadronic weak interactions like π and neutron decay are actually smaller than if universality held strictly by the factor $\cos^2\theta_c$

The <u>strangeness</u>-<u>changing</u> weak interactions (like K decay)



$\sim \sin\theta_c$

have rates suppressed by $\sin^2\theta_c$. Experimentally, $\sin^2\theta_c \sim .048-.051$, so strangeness-changing processes are significantly "Cabibbo suppressed"

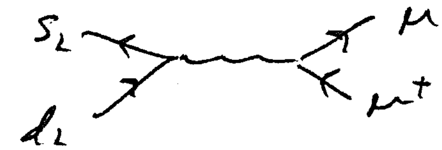
<u>Neutral Currents:</u>

Remember that the Z^0 couples to

$$\frac{e}{\cos\theta_W \sin\theta_W}\left(T^3 - \sin^2\theta_W Q\right)$$

The matrix $T^3 - \sin^2\theta_W Q$ obviously <u>commutes</u> with the unitary change of basis that diagonalizes the quark mass matrix, because <u>quarks of the same chirality and charge have the same value of T^3</u>. Thus, the structure of the neutral weak current is left intact by the change of basis; the current that couples to Z^0 is flavor <u>diagonal</u> in both the original basis and the mass eigenstate basis. The weak neutral current <u>preserves</u> flavor.

Hence, there is no tree graph contribution to, for example, $K_L \to \mu^+ \mu^-$



(Experimentally, has branching ratio $\sim 10^{-8}$) Indeed, observed suppression of this and other strangeness-changing neutral current effects made it possible to predict existence of charm quark c before it was discovered.

The logic goes like this: Cabibbo mixing is observed, so we know that weak eigenstates and mass eigenstates do not coincide. To avoid inducing strangeness-changing current coupling to Z^0, we must have same value of T^3 for d and s. Hence $T^3 = -\frac{1}{2}$, and s is in a weak $SU(2)_L$ multiplet with a $T^3 = +\frac{1}{2}$ partner -- the charm quark.

This trick for avoiding strangeness-changing neutral currents was invented by Glashow-Iliopoulos-Maiani in 1970, and is called the GIM mechanism.

3 (or more) Generations

This "mixing" of flavors in the charged weak currents occurs for any number of generations $N \geq 2$. Suppose

$$Q_L \quad U_R \quad D_R$$

have N flavor components, all transforming identically under $SU(2)_L \times U(1)_A$. Then the mass matrices M_U and M_D are $N \times N$ matrices, and the matrix

$$V = V_{LU}^{\dagger} V_{LD}$$

is a unitary $N \times N$ matrix. Let us count the number of free parameters in this matrix V, that cannot be absorbed by redefining phases of the quark fields.

A unitary $N \times N$ matrix V is specified by N real parameters, but in

$$\overline{U}_L \gamma^\mu V D_L,$$

$(2N-1)$ phases in V can be varied by adjusting the phases of the N up quarks and the N down quarks (Rotating U_L and D_L by the same overall phase has no effect on V.) So there are

$$N^2 - (2N-1) = (N-1)^2$$

free parameters in V that are actually measurable.

For three generations (N=3), then, there are 4 parameters. 3 are Cabibbo-like mixing angles (the most general real 3x3 unitary matrix has 3 angles) and the 4th is a phase. This unremovable phase is the origin of the CP violation in the standard model, something I do not have time to discuss here.

These 4 parameters that specify the flavor mixing in the three-generation standard model are called the Kobayashi-Maskawa parameters.

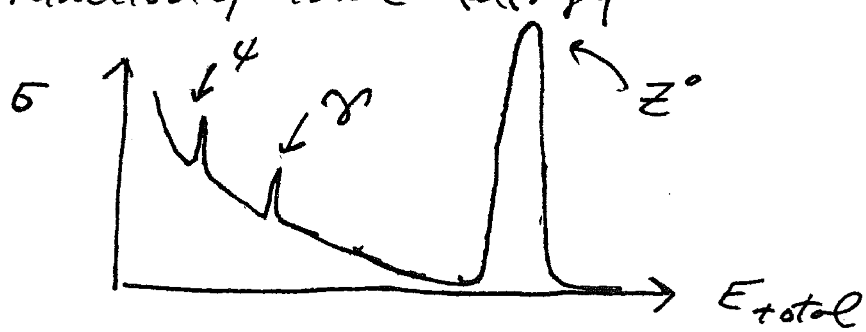
Physics at the Z^0

Let's briefly overview the near-term prospects for testing and exploring the standard model experimentally.

Here is a rough sketch of the cross section

$$\sigma(e^+e^- \to \text{hadrons})$$

as a function of total energy:



Interesting physics has been done by sitting on the ψ and Υ peaks (SPEAR, CESR, DORIS). Recently, PEP and PETRA have been trying to do physics in the deep valley, but it is tough going. It is clear that we should strive to do physics next on top of the Z^0 peak -- from the mountaintop we should have a clearer view.

Two "Z^0 factories" are being prepared:

SLC (SLAC Linear Collider) -- Fall '87
LEP (Large electron-positron storage ring at CERN) -- '89

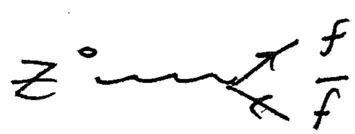
(when data expected)

What might we learn from these experiments?

<u>The Z^0 Peak</u>

To begin, let's estimate the width of the Z^0, and the factor by which the cross section is enhanced on resonance.

The Z^0 decays to fermion pairs by



+ higher order.

In a homework problem, you computed

$$\Gamma_{\nu\bar{\nu}} = \Gamma(Z^0 \to \nu_e \bar{\nu}_e) = 180 \text{ MeV}$$

(averaged over Z^0 spin) Since the coupling to the Z^0 is proportional to

$$T^3 - \sin^2\theta\, Q$$

we see that the decay rates to different fermion species are in the proportions (neglecting all fermion masses)

$$\begin{array}{llr}
\nu\bar{\nu}\,(L): & \left(\tfrac{1}{2}\right)^2 & = .25 \\
e^+e^-\,(L+R): & \left(-\tfrac{1}{2}+\sin^2\theta\right)^2 + \sin^4\theta & = .13 \\
\bar{u}u\,(L+R): & 3\left[\left(\tfrac{1}{2}-\tfrac{2}{3}\sin^2\theta\right)^2 + \tfrac{4}{9}\sin^4\theta\right] & = .44 \\
\bar{d}d\,(L+R): & 3\left[\left(-\tfrac{1}{2}+\tfrac{1}{3}\sin^2\theta\right)^2 + \tfrac{1}{9}\sin^4\theta\right] & = .56
\end{array}$$

(factor of 3 from counting quark colors) (for $\sin^2\theta = .22$)

Therefore, summing over all fermions in a generation, we have

$$\Gamma_{Gen} = \frac{.25+.13+.44+.56}{.25}\,\Gamma_{\nu\bar{\nu}} = 5.7\,\Gamma_{\nu\bar{\nu}} = 1.0 \text{ GeV}$$

For 3 generations, but <u>excluding</u> the <u>top</u> quark, we have

$$\Gamma_{tot} = 3.0 \text{ GeV} - \frac{.44}{.25}\Gamma_{\nu\bar{\nu}} = 2.7 \text{ GeV}$$

To summarize:

Z^0 width: $\Gamma_{tot} = 2.7$ GeV

Branching Fractions:

$$B_{\nu_e \bar{\nu}_e} = \frac{\Gamma_{\nu_e \bar{\nu}_e}}{\Gamma_{tot}} \cong .068$$

$$B_{e^+e^-} = .035$$

$$B_{\bar{u}u} = .12$$

$$B_{\bar{d}d} = .15$$

By what factor is the cross section enhanced on resonance? Recall the calculation on page (5.54):

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} \sim \frac{88 \text{ nanobarns}}{s \ (\text{GeV}^2)}$$

(through virtual photon)

At $s = (93 \text{ GeV})^2$, this would be

$$\sigma = .01 \text{ nb}.$$

Let's normalize the cross section at the Z^0 peak to this cross section.

Compare the Z^0 and γ couplings:

Since Z^0 couples to

$$\frac{e}{\cos\theta \sin\theta}(T^3 - \sin^2\theta \, Q)$$

and γ couples to e,

coupling of Z^0 to $f\bar{f}$ relative to coupling of γ to $f\bar{f}$ is

$$\frac{1}{\cos\theta\sin\theta} \quad \frac{T^3 - \sin^2\theta Q}{Q}.$$

The photon propogator is $\frac{1}{q^2}$.

But Z^0 propogator is $\frac{1}{q^2 - \mu_Z^2 + i\Gamma\mu_Z}$.

(See page 2.82.) So on resonance, these are in the ratio $(q^2 = \mu_Z^2)$

$$\left|\frac{Z^0 \text{ propogator}}{\gamma \text{ propogator}}\right|_{\text{on res}} = \left(\frac{\mu_Z}{\Gamma}\right)$$

Γ — total width

We conclude that

$$\left.\frac{\sigma(e^+e^- \to Z^0 \to f\bar{f})}{\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)}\right|_{\text{on res}} = \frac{(L_e^2 + R_e^2)(L_f^2 + R_f^2)}{4(\sin\theta\cos\theta)^4}\left(\frac{\mu_Z}{\Gamma}\right)^2$$

-- where L_f R_f are $T^3 - \sin^2\theta Q$ for LH and RH f respectively (the values of $L_f^2 + R_f^2$ for the various fermion species were tabled above.) The factor of 4 occurs in denominator because both LH and RH e's and μ's couple to γ with unit strength.

We have

$$\left.\frac{\sigma(e^+e^-\to Z^0\to f\bar{f})}{\sigma(e^+e^-\to\mu^+\mu^-)}\right|_{\substack{\text{on}\\ \text{res}}} = (L_f^2+R_f^2)\,\frac{.13}{4(.22)^2(.78)^2}\left(\frac{93}{2.7}\right)^2$$

$$\cong 1300\,(L_f^2+R_f^2)$$

If we sum $L_f^2+R_f^2$ for 3 generations, excluding top, we obtain 3.70, so

$$\left.\frac{\sigma(e^+e^-\to Z^0\to \text{all})}{\sigma(e^+e^-\to\mu^+\mu^-)}\right|_{\substack{\text{on}\\ \text{res}}} \cong 4800$$

Including only quarks

$$\sum(L_q^2+R_q^2) = 2.56\,,$$

so

$$\left.\frac{\sigma(e^+e^-\to Z^0\to \text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}\right|_{\substack{\text{on}\\ \text{res}}} \cong 3300$$

(excludes hadrons from τ decay)

Measurement of Z^0 Mass and Width:

By simply measuring the Z^0 mass, SLC will improve the experimental determination of $\sin^2\theta_W$ by over an order of magnitude. This measurement is expected to be limited by systematic effects to an accuracy

$$\Delta M_Z \sim 45 \text{ MeV},$$

or $$\frac{\Delta M_Z}{M_Z} \sim .0005$$

Since $$M_Z = \frac{e}{2\sin\theta\cos\theta}\left(\sqrt{2}\,G_F\right)^{-\frac{1}{2}}$$

$$\frac{dM_Z}{d\sin^2\theta} = M_Z \, \frac{1}{2} \, \frac{1-2\sin^2\theta}{\sin^2\theta\cos^2\theta} \sim 1.63\, M_Z$$

(for $\sin^2\theta = .22$)

So
$$\Delta \sin^2\theta \sim (1.63)^{-1}\,\frac{\Delta M_Z}{M_Z} \sim .0003$$

This is an improvement by about a factor of 30 over the best values of $\sin^2\theta$ obtain from low energy neutral current experiments.

Systematic effects will limit the accuracy of the measurement of the Z^0 width to

$$\Delta\Gamma \sim 35 \text{ MeV}.$$

This is a very important measurement that will give us information about the fourth generation (if any) and beyond. We've seen that each flavor of neutrino contributes

$$\Gamma_{\nu\bar{\nu}} = 180 \text{ MeV}$$

to the Z^0-width. So the measurement of the width of the Z^0 resonance will allow us to count the number of light neutrinos.

Polarization Asymmetry

It is important to subject the standard model to high precision tests; this is a good way to uncover new physics. Is there another way to measure $\sin^2\theta$, other than the Z^0 mass, which is of comparable accuracy? If so, the requirement that these two measurements agree provides a high precision test.

There is another good way to measure $\sin^2\theta$ — in the polarization asymmetry of $e^+e^- \to Z^0 \to \ell\bar{\ell}$. (This measurement is possible at SLC, which will have polarized e^+e^- beams, but not at LEP, which will have unpolarized beams.)

Since the current that couples to Z^0 is proportional to

$$\bar{e}_L \gamma^\mu e_L \left(-\tfrac{1}{2} + \sin^2\theta\right) + \bar{e}_R \gamma^\mu e_R \sin^2\theta + \cdots,$$

we see that

$$\left.\frac{\sigma_L(e^+e^- \to \ell\bar{\ell})}{\sigma_R(e^+e^- \to \ell\bar{\ell})}\right|_{\text{on res.}} = \frac{\left(-\frac{1}{2} + \sin^2\theta\right)^2}{\sin^4\theta} \sim 1.62$$

(for $\sin^2\theta = .22$)

Furthermore

$$\frac{d}{d\sin^2\theta}\left(\frac{\sigma_L}{\sigma_R}\right) = \frac{1}{\sin^4\theta}\left(1 - \frac{1}{2\sin^2\theta}\right) \sim -26$$

While σ_L/σ_R cannot be measured nearly as accurately as μ_Z, it is a much more sensitive function of $\sin^2\theta$,

$$\Delta \sin^2\theta \sim (26)^{-1}\, \Delta\left(\frac{\sigma_L}{\sigma_R}\right).$$

Thus σ_L/σ_R need only be measured to 1% accuracy to obtain a determination of $\sin^2\theta$ comparable in precision to that from μ_Z. (A 1% systematic uncertainty in polarization measurements may be achievable.)

Front-Back Asymmetry

The parity violation in the couplings of the Z can be measured without polarized beams as a "front-back asymmetry" of the differential cross section. (This could be done at LEP, but does not determine $\sin^2\theta$ so precisely.)

To see how this works, let us redo the calculation of $\sigma(e^+e^- \to \mu^+\mu^-)$ on page 5.54 ff for polarized electrons and muons. E.g. consider

$$e^+_L e^-_L \to \gamma \to \mu^+_L \mu^-_L$$

the only change from what we computed before is that the spinor traces now give (if we ignore the fermion masses)

$$\mathrm{tr}\left[\not p_1 \gamma^\mu \not p_2 \gamma^\nu \tfrac{1}{2}(1-\gamma_5)\right]\mathrm{tr}\left[\not p_1' \gamma_\mu \not p_2' \gamma_\nu \tfrac{1}{2}(1-\gamma_5)\right]$$

$$= 4\left(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \eta^{\mu\nu} p_1\cdot p_2 - i\epsilon^{\alpha\mu\beta\nu} p_{1\alpha} p_{2\beta}\right)$$
$$\left(p_1'^\mu p_2'^\nu + p_1'^\nu p_2'^\mu - \eta^{\mu\nu} p_1'\cdot p_2' - i\epsilon_{\gamma\mu\delta\nu} p_1'^\gamma p_2'^\delta\right)$$

$$= 4\Big[\underbrace{2(p_1\cdot p_1')(p_2\cdot p_2') + 2(p_1\cdot p_2')(p_2\cdot p_1')}_{\text{what we had before}}$$
$$\underbrace{-2\left(\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right) p_{1\alpha} p_{2\beta} p_1'^\gamma p_2'^\delta}_{\text{new term}}\Big]$$

$$= 8\left[(p_1\cdot p_1')(p_2\cdot p_2') + (p_1\cdot p_2')(p_2\cdot p_1')\right.$$
$$\left. - (p_1\cdot p_1')(p_2\cdot p_2') + (p_1\cdot p_2')(p_2\cdot p_1')\right]$$

In cm frame, and defining E, θ as on page (3.55), this is

$$8E^4\Big[\underbrace{(1-\cos\theta)^2 + (1+\cos\theta)^2}_{\text{old term}} \underbrace{-(1-\cos\theta)^2 + (1+\cos\theta)^2}_{\text{new term}}\Big]$$

So, for LH e's and μ's, the differential cross section becomes

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4s}\quad(1+\cos\theta)^2$$

We got the same for $e^+_R e^-_R \to \mu^+_R \mu^-_R$, but in

$$e^+_R e^-_R \to \mu^+_L \mu^-_L$$
$$e^+_L e^-_L \to \mu^+_R \mu^-_R,$$

the new term has a different relative sign. So

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{e^+_L e^-_L \to \mu^+_L \mu^-_L \\ e^+_R e^-_R \to \mu^+_R \mu^-_R}} = \frac{\alpha^2}{4s}\,(1+\cos\theta)^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{e^+_L e^-_L \to \mu^+_R \mu^-_R \\ e^+_R e^-_R \to \mu^+_L \mu^-_L}} = \frac{\alpha^2}{4s}\,(1-\cos\theta)^2$$

When we sum together all 4 processes, we recover our old answer (in the massless limit):

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{e^+e^-\to\mu^+\mu^- \\ \text{spin ave.}}} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right)$$

Notice that the LL and RR differential cross sections vanish at $\cos\theta = -1$, and the LR and RL differential cross sections vanish at $\cos\theta = 1$. This behavior is easy to interpret. Note that e^+_L denotes the antiparticle of the Weyl fermion e^-_L; thus e^+_L has helicity $+\frac{1}{2}$ while e^-_L has helicity $-\frac{1}{2}$. So the angular momenta

$$e_L^+ \overset{\Rightarrow}{\longrightarrow} \qquad \overset{\Rightarrow}{\longleftarrow} e_L^-$$

of e_L^+ and e_L^- along the beam axis add up to $+1$. But a $\mu_L^+ \mu_L^-$ pair with $\cos\theta = -1$ has angular momentum -1 along the same axis, so the process is forbidden. The same argument applies to the other cases.

Now, the spin sums described above also occur in the Z^0 exchange diagram. On the Z^0 resonance, (where we may neglect the γ exchange diagram) the spin-averaged differential cross section is proportional to

$$\left(\frac{d\sigma}{d\Omega}\right)_{\substack{e^+e^- \to f\bar{f} \\ \text{spin ave}}} \quad \left[(R_e^2 + L_e^2)(R_f^2 + L_f^2)(1+\cos^2\theta) + 2(R_e^2 - L_e^2)(R_f^2 - L_f^2)\cos\theta \right]$$

The second term contributes to front-back asymmetry, but doesn't contribute to the total cross section. If we define an asymmetry by

$$A = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma_{total}},$$

then

$$A = \frac{2(R_e^2 - L_e^2)(R_f^2 - L_f^2)}{(8/3)(R_e^2 + L_e^2)(R_f^2 + L_f^2)}$$

For example, the asymmetry in $e^+e^- \to \mu^+\mu^-$ is

$$A(e^+e^- \to \mu^+\mu^-) = \frac{3}{4}\left[\frac{(\frac{1}{2}-\sin^2\theta)^2 - \sin^4\theta}{(\frac{1}{2}-\sin^2\theta)^2+\sin^4\theta}\right]^2 \sim .04$$

(for $\sin^2\theta = .22$)

This is rather small because the parity violation in the lepton coupling is small (it would vanish identically for $\sin^2\theta = \frac{1}{4}$).

Looking for New Physics

Various new particles might conceivably be seen by experiments that observe the decay modes of the Z^0

- **The Top Quark**

Can be seen in $Z^0 \to t\bar{t}$ if top is light enough. This is easy to identify when t decays semileptonically:

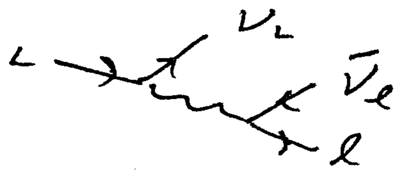
(Jets are seen with non collinear leptons.)

- **New ν's**

As already mentioned $Z^0 \to \nu\bar{\nu}$ will increase the width of the Z^0 by 180 MeV for each new light neutrino species.

- **New Heavy Leptons**

$Z \to L^+ L^-$ has a branching fraction 3% × (phase space suppression). It can be identified when L decays leptonically:



- **The Higgs Boson**

Has coupling $\mu_Z^2 Z_\mu Z^\mu \left(\frac{\phi'}{v} + \frac{1}{2}\left(\frac{\phi'}{v}\right)^2 \right)$ as described earlier. Can be seen through

$Z \to \phi' e^+ e^-$

ϕ' decays to heaviest available fermion pair, since its Yukawa coupling is proportional to mass. Of course, this decay doesn't occur if ϕ' is too heavy. (Then we may have to wait for e.g. $\phi' \to W^+ W^-$ at the SSC.)

The Standard Model -- An Appraisal

We've been concentrating almost solely on the electroweak interactions, but it turns out that strong interactions can also be well-described by a gauge theory. This theory, quantum chromodynamics (QCD), has gauge group

$$G = SU(3)_{color}$$

and both LH and RH quarks transform as triplets under $SU(3)_c$.

All of particle physics as we know it, then, is well accounted for by a "standard model" with gauge group

$$G_{\substack{\text{standard}\\ \text{model}}} = SU(3)_{color} \times SU(2)_L \times U(1)_Y$$

Let us count up the elementary degrees of freedom of this theory.

Degrees of Freedom:

First, the fermions. Each generation of quarks and leptons transforms as the representation of the gauge group:

$$\underset{\text{doublet})}{(SU(2)_L} \Big[\underbrace{\begin{pmatrix} u_{L1} & u_{L2} & u_{L3} \\ d_{L1} & d_{L2} & d_{L3} \end{pmatrix}^{1/6}}_{(SU(3)_c \text{ triplet})} \; \begin{matrix} (u_{R1}\, u_{R2}\, u_{R3})^{2/3} \\ (d_{R1}\, d_{R2}\, d_{R3})^{-1/3} \end{matrix} \; \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}^{-\frac{1}{2}} \; e_R^{-1}$$

(The superscripts denote $U(1)_Y$ charge assignments.)

This is altogether 15 two-component Weyl fermions per generation. Three generations are known, so that is 45 elementary fermion species.

(A different way to do this counting is to count each irreducible representation of G as a different degree of freedom. By this reckoning there are 5 species per generation, or 15 altogether.)

The number of gauge fields is

$$\underset{SU(3)_C}{8} + \underset{SU(2)_L}{3} + \underset{U(1)_Y}{1} = 12$$

-- Or, if one prefers, three different irreducible reps, corresponding to the 3 "factors" of the gauge group.

Finally, there is the Higgs boson:

$$\begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}^{-\frac{1}{2}}$$

The doublet contains 4 real scalars, but it might be preferable to count this as just one degree of freedom, since 3 are eaten by gauge bosons, leaving only ϕ!

The total number of elementary particles in the 3 generation standard model may thus be said to be---

45 fermions + 12 vector bosons + 1 scalar = 58.

Parameters:

Next, let us count the number of (in principle measurable) free parameters in the model.

Gauge couplings:

g_1, g_2, g_3 — 3 parameters

(for $U(1)_Y$, $SU(2)_L$, $SU(3)_c$)

Fermion Masses and Mixings:

Lepton Masses - 3
Quark Masses - 6
KM Parameters - 4

$\Big]$ -- 13 parameters

Scalar Masses:

1 ϕ' Mass -- 1 parameter

Total: 17 parameters

Note: $v = (\sqrt{2} G_F)^{-\frac{1}{2}}$ sets the mass scale, and does not qualify as a dimensionless parameter. Higgs self coupling, Yukawa couplings, M_W, M_Z, etc can all be expressed in terms of above and v. (When we include nonperturbative effects, two new parameters enter, called θ parameters. Let's not worry about those.)

So we have a model, with 58 elementary particles and 17 adjustable free parameters, that seems to fit all known particle phenomenology. Does this mean that particle physics is over?

One hopes not. And it is clear that there are good reasons for believing that the standard model is not the final word. We should hope for a deeper theoretical understanding. And, in fact, one may be justified in believing that there is radically new physics awaiting discovery that will be accessible to forseeable accelerators.

When the top quark and the Higgs mass are discovered, we will have pinned down the 17 parameters. But many theoretical questions will remain. For example...

<u>Some Questions</u>:

- Why is the gauge group $SU(3) \times SU(2) \times U(1)$?
- Why are the fermions in the particular representation of the gauge group

$$(3,2)_L^{1/6} + (\bar{3},1)_R^{2/3} + (\bar{3},1)_R^{-1/3} + (1,2)_L^{-1/2} + (1,1)_R^{-1}$$

?

- Why are there <u>three</u> generation? (if, in fact, there are only three)

- What determines the values of the 17 parameters?
- How does <u>gravity</u> fit in with the other interactions?

<u>Remark</u>:

One of the above questions can actually be partially answered within the framework of the standard model (without invoking new physics). Concerning the representation content of a generation of fermions, one might ask, given how quarks and leptons transform under $SU(3)_c \times SU(2)_L$, how are their crazy $U(1)_Y$ charges determined? This question has an essentially unique answer if we demand anomaly <u>cancellation</u>, that is, that quantum effects do not destroy gauge invariance. It is also interesting that, because of the need for cancellation of anomalies, a theory of quarks alone or leptons alone does not make sense (is not gauge invariant). The quarks need the leptons and vice-versa.

All of the above questions are deep and fundamental. Any truly comprehensive theory of Nature should address these questions. But, unfortunately, there is no guarantee that the answers to these questions will involve physics that we can hope to probe in forseeable experiments. (For example, there may be a fundamental theory involving new physics at the Planck mass $m_P \sim 10^{19}$ GeV, with no new physics at lower energy scales. If so, we'll never answer any of these questions by doing experiments.) But there is one more question that has a different character, because the fact that it remains unanswered by the standard model strongly suggests that there will be new physics at energies not far above the W and Z masses. This question is --

- What is the origin of the mass scale $v \sim 250$ GeV of the standard model?

If we wish to phrase this as a question about a dimensionless number, we note that $m_P \sim (G_{Newton})^{-\frac{1}{2}} \sim 10^{19}$ GeV is another fundamental scale in physics, and so we ask.

- What determines $v/m_P \sim 10^{-17}$?

Since this takes the form of seeking an explanation for a very small number, it is sometimes called the "hierarchy problem." Why are the characteristic mass scales of the weak and gravitational interactions vastly different?

Now, to appreciate why this question encourages us to believe in accessible new physics, recall the claim back on page 2.125: "Elementary scalars are unnatural". There, we argued that, if there is physics with a characteristic mass scale Λ, and an effective field theory with cutoff Λ describes physics below this scale, then we can arrange for the effective field theory to contain an elementary scalar with mass

$$m^2 << \Lambda^2$$

only by "fine-tuning" the bare scalar mass to an accuracy of order

$$\left(\frac{m^2}{\Lambda^2}\right)$$

Since such fine-tuning seems intrinsically implausible, we are led to regard elementary scalars with $m^2 << \Lambda^2$ as unlikely to exist.

Because the scale v is actually determined by a parameter in the Lagrangian of the standard model that is the renormalized Higgs mass ($m_H^2 \sim -\lambda v^2$), we suspect that

The standard model indicates that new physics is needed at a scale Λ not far above v. This point is enormously important. It is the best argument we have that new physics awaits discovery at the TeV scale. We may indeed have the privilege in the coming decades of unraveling new physics that goes beyond the standard model.