Physics 230

Exercises
Due: Wednesday, 17 May 2000

7.1 Perturbative Wilson loop

In four-dimensional Yang-Mills theory with action

$$S = \int d^4x \left(\frac{1}{4} F^{\mu\nu a} F^a_{\mu\nu} \right) , \tag{1}$$

calculate to order g^2 the vacuum expectation value $\langle W(C) \rangle$ of the Wilson loop operator

$$W(C) \equiv \mathrm{tr} \left(P \exp \left(ig \oint_C T^a A^a_\mu dx^\mu \right) \right) , \tag{2}$$

with the generators normalized so that $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$. Consider a rectangular loop in Euclidean space with sides R and T, assume $T \gg R$, and retain only the term proportional to T/R. Interpret the result.

7.2 Instantons in (1+1)-dimensional Yang-Mills Theory

In (1+1)-dimensional spacetime, consider Yang-Mills theory with gauge group $G = SU(N)/Z_N$, and Higgs field in the adjoint representation of G. Choose a Higgs potential such that all gluons acquire masses from the perturbative Higgs mechanism.

a) Does this model have instanton solutions? Is there a θ-parameter? What are the allowed values of θ? What is the physical interpretation of θ?

b) In the dilute-instanton-gas approximation, calculate the θ-dependent string tension and interpret the result.

7.3 Duality and Higgs exotica

Consider an $SU(N)/Z_N$ gauge theory on a 3-torus. Let $(\vec{e}, \vec{m})$ denote the Z_N electric and magnetic fluxes associated with the three fundamental nontrivial cycles of the torus, and let $Z(\vec{e}, \vec{m})$ denote the partition function of the theory in the flux sector $(\vec{e}, \vec{m})$. These quantities obey the *duality relation*:

$$Z(\vec{e}, \vec{m}) = \frac{1}{N^2} \sum_{k_1,k_2,l_1,l_2} \exp\left[\frac{-2\pi i}{N}(k_1 e_1 + k_2 e_2 - l_1 m_1 - l_2 m_2)\right] \cdot Z(l_1, l_2, e_3; k_1, k_2, m_3) \ . \tag{3}$$

If the theory has a mass gap, then in the limit of a large box, each $Z(\vec{e}, \vec{m})$ must have either the value 1 ("light flux"), or 0 ("heavy flux"). In class we found for $N = p$ prime that the only solutions to the duality relation that are compatible with rotational invariance are electric confinement ($(0, \vec{m})$ is light) and magnetic confinement ($(\vec{e}, 0)$ is light).

a) Consider $N = p_1 p_2$ where p_1 and p_2 are prime. Find an exotic solution to the duality relation (neither electric confinement or magnetic confinement).

b) Interpret the solution found in (*a*) in terms of the Higgs mechanism. That is, propose a pattern of symmetry breaking, and Higgs fields driving the symmetry breakdown, which would give rise to these light and heavy fluxes.