Physics 236c
Relativity

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Spring, 1990

Topic: Quantum field theory in curved spacetime

Prerequisites:

Basic General Relativity (e.g. causal structure of extended Schwarzschild, de Sitter; Penrose diagrams)

Basic Quantum Field Theory (e.g. quantization of free scalar field, path integral)

Book (Recommended):

N. D. Birrell + P.C.W. Davies, Quantum fields in curved space, Cambridge, 1982. $19.95 in paperback

covers roughly the same material at roughly the same level as the course.

Requirements: Problem Sets

Grading: Pass/Fail

Graders: Martin Bucher
Fernando Echevarria
Steinn Sigurdsson

This course is inspired by a suggestion due to Kip Thorne, that devoting the final quarter of a full-year relativity course to quantum field theory on a curved spacetime background would be a Good Thing.
The main objective of the course will be to understand the semiclassical theory of black hole radiance.

Introduction

A typical first year graduate student may take a year-long course on general relativity and a year-long course on advanced (relativistic) quantum mechanics. Most likely, he hears little about quantum theory in the GR course, and little about gravity in the QFT course. Why?

There are practical reasons. Experts in GR are often not experts in QFT, and vise versa. Most interesting physics applications of GR concern phenomena on large distance scales where QFT has little relevance — structure and evolution of universe, relativistic astrophysics and astrophysical black holes, sources of gravitational radiation. Likewise, the interesting physics applications of QFT typically concern phenomena at very short distances — e^+e^- or pp collisions. These applications fail to provide a context in which gravity and quantum theory need be considered simultaneously.

In what sort of context are GR and QFT simultaneously relevant?

A first hint was actually provided in 1899 by Planck, who noticed that the fundamental constant $\hbar$, along with c and Newton's constant G

allow us to construct fundamental units of distance, time, energy, etc...

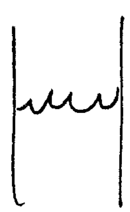
Distance: $L_P = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} \sim 10^{-33}$ cm

Time: $T_P = \frac{L_P}{c} = \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}} \sim 10^{-43}$ sec

Energy: $E_P = \frac{\hbar}{T_P} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} \sim 10^{19}$ GeV

Just on dimensional grounds, then, one expects that gravity and quantum theory are relevant for phenomena at distance $\sim L_P$ or energy $\sim E_P$.

To be more concrete, consider the gravitational contribution to hard (large-angle) scattering between relativistic particles. Merely



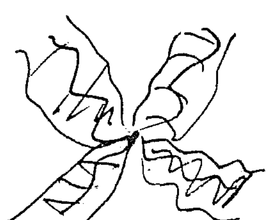
on dimensional grounds, the cross section is of order

$$\sigma \sim G^2 E^2 \qquad (\hbar = c = 1)$$

This becomes appreciable (comparable to the "unitarity limit" $\sim \frac{1}{E^2}$) for $E \sim E_P \sim 10^{19}$ GeV. At such energies quantum corrections higher order in $\hbar$ become important. These corrections turn out to be so sensitive to the nature of quantum fluctuations with

$\omega \gtrsim E_P$ that the calculability of the theory breaks down. (Einstein gravity is unrenormalizable.) This breakdown of calculability at $E \sim E_P$ is an indication that Einstein gravity must be replaced by a more complete underlying theory. — Superstring theory?

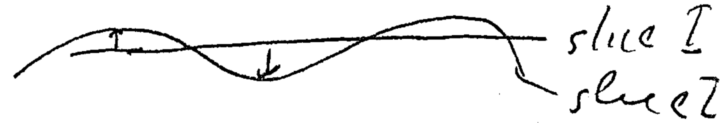
The breakdown of Einstein gravity at $L \sim L_P$ is more profound than say the breakdown of Fermi's weak interaction theory at $E \sim 300$ GeV. For the dynamical variable of quantum gravity is the spacetime metric itself. Large Quantum gravitational fluctuations may mean that the whole notion of a spacetime is breaking down. At the very least, it is obscure how to formulate microscopic causality for a theory in which the edge of the light cone is broad and fuzzy.



Quite aside from the ambiguities that afflict quantum gravity at "short-distances" there are deep conceptual questions concerning the interpretation of a quantum theory of gravity — these stem from the general covariance of the theory. In general relativity, the coordinates that parametrize spacetime have no invariant significance, and some are free to reparametrize the spacetime locally. When we formulate quantum mechanics, we must (arbitrarily) choose a time coordinate t, and construct a Hamiltonian H that generates time evolution. Now we should distinguish two types of reparametrizations, which have quite distinct physical consequences:

1) We can reparametrize <u>each</u> time slice t = constant. This is an example of gauge invariance — it is a redundancy in how the metric ^3g of the time slice parametrizes the quantum states, like the redundancy in how the vector potential A_μ of electrodynamics parametrizes states. The "wave function" $\psi(^3g)$ must be regarded as a functional of the 3-<u>geometry</u> represented by the metric, not the metric itself.

2) We can reparametrize time, or choose a new set of time slices.



Because we have the freedom to move the slice forward or backward locally, the quantum theory will not tolerate <u>any</u> time-dependence...

$$i\frac{\partial}{\partial t}\psi = H\psi = 0$$ -- Wheeler-DeWitt equation

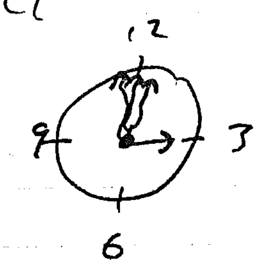
So, naively, the theory has no dynamics, at least with respect to the extrinsic parameter time. To formulate dynamics, we must identify an intrinsic time. We ask not

"what is $P(x,t)$, the probability that the electron is at x at 12 noon."

but

"what is $P(x/t)$, the conditional prob that the electron is at x if the clock says t."

The change is not trivial, because if time is the reading of a clock that is itself a quantum-mechanical dynamical variable, then time itself is subject to quantum fluctuations.



Without a meaningful extrinsic time, it is not clear how we are to construct a self consistent probability interpretation of quantum mechanics. What we ordinarily do is choose a complete set of commuting observables and consider states $|\lambda, t\rangle$ that are eigenstates of all these observables at time t

then $P(\lambda, t) = |\langle \lambda, t | \psi \rangle|^2$ is the prob distribution for a measurement of the observables. It satisfies "conservation of probability"

$$\sum_\lambda P(\lambda,t) = \sum_\lambda \langle\Psi|\lambda,t\rangle\langle\lambda,t|\Psi\rangle = \langle\Psi|\Psi\rangle = 1$$

– because the observables are complete

Because of the "problem of time" in general relativity, it is not clear how to construct a complete set of observables that obey conservation of probability. There is a "semiclassical" limit in which fluctuations in time can be ignored and the usual theory applies, but we would like quantum gravity to have an interpretation beyond semiclassical theory. (And what ensures that semiclassical behavior is ever attained?)

Thus, because of the problem of time, we do not yet understand whether quantum gravity has a fully consistent probability interpretation. This is a deep conceptual problem, quite independent of the short-distance problem.

But $E \sim E_P \sim 10^{19}$ GeV, $L \sim L_P \sim 10^{-33}$ cm seems so remote from all experience, should we really care about such issues. The motivation to understand quantum gravity comes from --

1) The problem of the initial conditions of the universe — "Quantum cosmology"

2) What is the final state of an evaporating black hole? (Does quantum mechanics break down?)

3) The hope that a better understanding of quantum gravity will elucidate physics in unexpected ways, including consequences more directly related to experiment

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Although we have argued that understanding quantum gravity is important, we will not be quite so ambitious in this course. In the hope of evading the conceptual quagmire described above, we will mostly confine our attention to $L \gg L_P$ and $E \ll E_P$.
In this regime, it is an excellent approximation to do quantum field theory, but treat gravity as classical.

E.g. in Z^0 decay $Z^0 \to \mu^+\mu^- + \text{hard graviton}$

gives a contribution suppressed by $\sim G M_Z^2 \sim 10^{-34}$ to the total Z width — Gravity is negligibly weak at low energy.

— The nonrenormalizability of quantum gravity does not alter this conclusion.

the large quantum fluctuations at $L \sim L_P$ decouple from the long distance physics — except for effects that can be absorbed into renormalization of a few free parameters. This decoupling of short wavelength from long wavelength physics is crucial — without it, quantum field theory, and physics, would not be possible at all.

In this course, we will focus on the effect of classical gravity on the quantum theory of other fields — In other words, we will consider quantum field theory on a spacetime background with curvature that is nonzero, but small in Planck units. The curvature significantly affects excitations and fluctuations of the fields on wavelengths comparable to the scale set by the curvature.

By considering this limit, we will not avoid all subtleties and ambiguities. For example, the notion of a particle is difficult to make precise on a curved background. Correspondingly, it will not always be obvious how to define a "vacuum" state of the quantum fields or how to construct a Hilbert space built on the vacuum. These are features not encountered in classical GR or in QFT on flat space, and we will need some ingenuity and pragmatism to deal with them.

Further subtleties arise when we consider the backreaction of the quantum fields on the spacetime geometry. It is simply not consistent to say that the quantum fields other than gravity fluctuate — for these fields are a source for gravity. In a zeroth order approximation we may regard $\langle \psi | T_{\mu\nu} | \psi \rangle$ as a source of back reaction, but this is reasonable only if the fluctuations in $T_{\mu\nu}$ are small enough so they will perturb the geometry only a little. And it is not easy to decide how the operator $T_{\mu\nu}$ appearing in the expectation value is to be constructed.

Our main objective in this course is to understand the classic applications of quantum field theory in curved spacetime

- Black Hole Radiation —
  the key to understanding the thermodynamics and entropy of black holes, and the (potential) loss of quantum coherence

- Quantum Fluctuations in de Sitter Space —
  Perhaps the origin of the primordial mass density fluctuations in the early universe that seeded galaxy formation, in the inflationary universe scenario.

In attempting to grasp the above applications, it will also be enlightening to consider—

- Rindler Space —
  the thermal radiation seen in the vacuum by a uniformly accelerated observer.

(In all these cases, we will be able to discern the essential features by considering free quantum fields with zero spin.)

The essential feature that all three examples have in common is: an event horizon

We will also consider the back reaction problem — how the quantum fields act as source for the geometry, and how $T_{\mu\nu}$ is to be defined as a quantum mechanical operator.