

Universal scaling laws for correlated decay of many-body quantum systems

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Quantum systems are inevitably open, continually exchanging energy and information with the surrounding environment. This interaction leads to decoherence and decay of quantum states. In complex systems, formed by many particles, decay can become correlated and enhanced. A fundamental question then arises: what is the maximal decay rate of a large quantum system, and how does it scale with its size? Computing this rate exactly is as hard as finding the ground-state energy of a generic spin Hamiltonian—a notoriously intractable problem. Here we exploit this correspondence to establish rigorous and general upper and lower bounds on the maximal decay rate. These bounds are universal, as they hold for a broad class of Markovian many-body quantum systems. For many physically relevant systems, the bounds are asymptotically tight, resulting in exact scaling laws with system size. Specifically, for large atomic arrays in free space, these scalings depend only on the dimensionality of the array and are insensitive to details at short length scales. The scaling laws set fundamental limits on the decay rates of all quantum states, shed light on the behaviour of generic driven-dissipative systems, and may ultimately constrain the scalability of quantum processors and simulators based on atomic arrays.

Understanding the quantum dynamics of far-from-equilibrium open many-body systems is a major frontier in physics. From a fundamental perspective, the interplay between energy pumping and dissipation allows for the emergence of phases that transcend the paradigms established by equilibrium statistical physics. Examples in quantum optics include the superradiant laser^{1,2} and the driven Dicke phase transition^{3–5}. From an applied standpoint, the full potential of quantum technologies—including quantum computing, quantum simulation and metrology—is realized only with large systems that remain coherent despite their coupling to a bath.

In systems formed by many particles, the always-present vacuum fluctuations mediate long-range dissipative interactions that cannot be switched off, inducing correlated decay that may increase with system size. Such decay processes are collectively enhanced if the particles are tightly packed. Correlated decay may, thus, become the ultimate source

of decoherence for many quantum technologies. For instance, it may alter the signal-to-noise ratio in metrology experiments such as atomic clocks or spin squeezing. Similarly, in large-scale quantum computers, it can lead to much shorter coherence times than the predicted timescales using independent noise models and may hinder quantum error correction^{6,7}. On the other hand, correlated decay is a critical requirement for other applications, such as the development of new light sources^{1,2,8}, the dissipative preparation of correlated many-body states^{9,10} or the protection of logical quantum information via dissipation^{11,12}.

Due to the exponential complexity associated with large quantum systems, exactly computing the largest decay rate is a formidable challenge. This problem remains unsolved except in trivial cases, such as permutationally symmetric models (for example, atoms coupled to a cavity) and non-interacting systems. In generic situations, finding the largest decay rate is as difficult as determining the ground state

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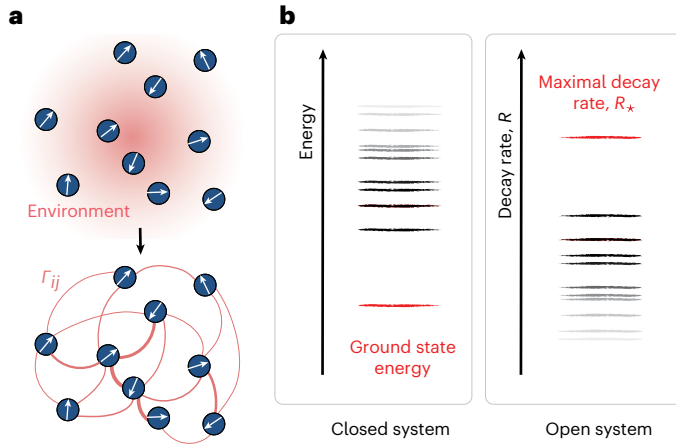


Fig. 1 | Generic qubit ensemble described as an out-of-equilibrium, open, many-body quantum system. a, In Markovian baths, integrating out the environment degrees of freedom yields a spin model with coherent and dissipative interactions. The dissipative couplings between N qubits are given by the decoherence matrix $\Gamma = (\Gamma_{ij})_{i,j=1}^N$. **b**, For a closed system, the ground state is the state of minimal energy. For an open system, finding the state with maximal decay rate (R_*) is analogous to finding the ground-state energy of a Hamiltonian.

of a general 2-local Hamiltonian, which is known to be a quantum Merlin–Arthur-complete problem—believed to be hard even for a quantum computer¹³. This complexity is compounded by the diversity of experimental platforms, with many candidate qubits (neutral atoms, molecules, ions, superconducting qubits, quantum dots, vacancy centres among others) and various mediators of their interactions (electromagnetic field and other bosonic collective excitations such as phonons, magnons and so on).

In this work, we find upper and lower bounds to the maximal decay rate by leveraging tools from Hamiltonian complexity theory^{14,15} and applying them in the context of out-of-equilibrium quantum dynamics. For a large class of systems, these bounds are asymptotically tight, thereby yielding scaling laws with system size that only depend on the spectral properties of the decoherence matrix Γ , whose dimension is linear in system size. The bounds are obtained by means of product-state ansatzes and, thus, imply that entanglement does not play any role in the scaling. Our results are formally rigorous and do not rely on mean-field approximations: although the bounds are derived using product states, they remain valid for any state (including entangled states), thereby going beyond previous approaches based on mean-field theory¹⁶ or approximate numerical methods^{17–19}.

We apply these tools to the specific case of ordered atomic arrays^{20–22} and lattices in free space²³, which have become an all-around platform for different quantum technologies, ranging from quantum computing²⁴ and quantum simulation^{25–27} to atomic clocks^{28–30} and spin squeezing^{31–33}. In the physically relevant regime of lattice constants similar to the resonance wavelength, the maximal decay rate scales as $\sim N^{\frac{3}{2} - \frac{1}{2D}}$, where D is the array dimensionality. This scaling law is universal as it does not depend on specific details of the array such as lattice geometry or atomic polarization and has implications in a broad set of problems ranging from quantum dynamics to metrology and quantum computation.

Theory background

A broad class of Markovian many-body open quantum systems of N qubits (Fig. 1a) is described by the Lindblad master equation

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H} + \hat{H}_{\text{local}}, \hat{\rho}] + \mathcal{D}_{\text{local}}(\hat{\rho}) + \sum_{i,j=1}^N \Gamma_{ij} \left(\hat{\sigma}_i^- \hat{\rho} \hat{\sigma}_j^+ - \frac{1}{2} \{ \hat{\sigma}_j^+ \hat{\sigma}_i^-, \hat{\rho} \} \right), \quad (1)$$

where $\hat{\sigma}_i^{\pm} = (\hat{\sigma}_i^x \pm i\hat{\sigma}_i^y)/2$ are the raising and lowering operators for qubit i . The first term describes coherent evolution, governed by an arbitrary qubit Hamiltonian $\hat{H}$ that commutes with the total excitation operator $\hat{n}_{\text{exc}} = \sum_i \hat{\sigma}_i^+ \hat{\sigma}_i^-$, and an arbitrary local Hamiltonian $\hat{H}_{\text{local}}$ that can be written as a linear combination of local Pauli operators (that is, each Pauli operator acts non-trivially on a constant number of qubits). The remaining terms describe dissipation, either local—in the form of a linear combination of Lindbladians $\mathcal{D}_{\text{local}}$ in which all Lindblad operators are arbitrary local Pauli operators—or collective, generated by Lindblad operators with non-local support that act on an extensive number of qubits. Precise definitions of $\hat{H}_{\text{local}}$ and $\mathcal{D}_{\text{local}}$, which model non-collective processes, are provided in the Methods.

Collective dissipative interactions are represented by the Hermitian decoherence matrix $\Gamma = (\Gamma_{ij})_{i,j=1}^N$. For the master equation to describe a physically valid evolution (that is, a completely positive and trace-preserving map), Γ must be positive semidefinite (that is, $\Gamma \geq 0$). This ensures non-negative eigenvalues $\{\Gamma_{\mu}\}$, which are physically interpreted as collective transition rates. Since $\Gamma \geq 0$, the spectral norm $\|\Gamma\|$ is equal to Γ_{max} , the largest collective transition rate (that is, the largest eigenvalue of the matrix). We define $\Gamma_0 = \sum_{i=1}^N \Gamma_{ii}/N$ to be the average individual decay rate.

We note that the above master equation encompasses a wide variety of physical scenarios, including arbitrary Hamiltonian interactions, coherent driving, diverse decoherence channels (such as incoherent driving, coupling to a finite-temperature reservoir, and dephasing) and disorder in Γ (Supplementary Section A.1 provides a detailed discussion). However, for the sake of simplicity, we omit the local terms $\hat{H}_{\text{local}}$ and $\mathcal{D}_{\text{local}}$ in the following discussion, a choice that we justify later.

The instantaneous decay rate of the many-body system, R , is computed as the expectation value of an ‘auxiliary’ (and Hermitian) Hamiltonian $\hat{H}_{\Gamma}$ (ref. 34), that is,

$$R = -\frac{d}{dt} \langle \hat{n}_{\text{exc}} \rangle \equiv \langle \hat{H}_{\Gamma} \rangle / \hbar \quad (2)$$

where

$$\hat{H}_{\Gamma} = \hbar \sum_{i,j=1}^N \Gamma_{ji} \hat{\sigma}_i^+ \hat{\sigma}_j^- = \hbar \sum_{\mu=1}^N \Gamma_{\mu} \hat{c}_{\mu}^{\dagger} \hat{c}_{\mu}, \quad (3)$$

and $\Gamma_{ji} = \Gamma_{ij}^*$. The last equality is achieved by means of collective jump operators $\hat{c}_{\mu} = \sum_{i=1}^N \alpha_i^{(\mu)} \hat{\sigma}_i^-$, with $(\alpha^{(\mu)})$ being the normalized eigenvectors of Γ (in this notation, the largest eigenvalue is $\Gamma_{\text{max}} \equiv \Gamma_1$). Generically, the auxiliary Hamiltonian $\hat{H}_{\Gamma}$ describes an XY model defined on a weighted interaction graph with a local transverse field. In the specific case where the interactions are mediated by the electromagnetic field, Γ is proportional to the electromagnetic Green’s function³⁵ and the decay rate is exactly equal to the photon emission rate integrated over all emission angles.

Lower and upper bounds

Our goal is to set the theoretical limits on the maximal decay rate R_* , the maximum value of R over all possible many-body quantum states. As demonstrated in Supplementary Section A.2, R_* also provides upper bounds on the rates of change for general observables. For example, the average coherence $N^{-1} \sum_j \langle \hat{\sigma}_j^- \rangle$ changes at a rate at most $\sqrt{2R_* \Gamma_0}$, which captures the collectively enhanced decoherence due to correlated decay.

Computing R_* amounts to calculating the spectral radius of $\hat{H}_{\Gamma}$ (since $\hat{H}_{\Gamma} \geq 0$) or, equivalently, the ground-state energy of $-\hat{H}_{\Gamma}$ (Fig. 1b). Finding the exact energy is expected to be hard¹³, except in two limiting cases. For non-interacting qubits (with $\Gamma_{ij} = \Gamma_0 \delta_{ij}$), $R_* = N\Gamma_0$. In the Dicke limit (that is, with all-to-all interactions such that $\Gamma_{ij} = \Gamma_0 \forall i, j$),

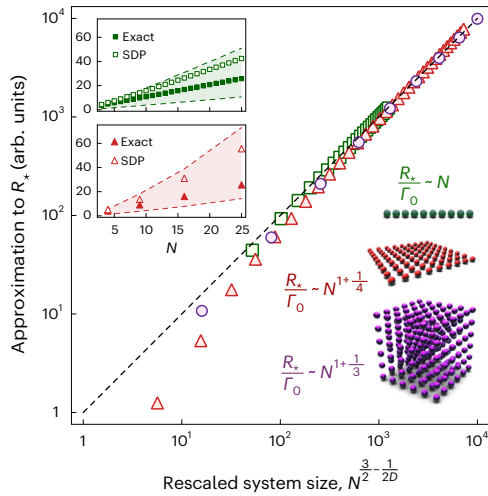


Fig. 2 | Scaling with system size of the numerical approximation for R_* . An approximation to the maximal decay rate R_* is given by the SDP solution R_{SDP} for different lattice dimensionalities with lattice constant $d = 0.4\lambda_0$. For 1D ($\square$) and 2D ($\triangle$) arrays, the atoms are perpendicularly polarized; for the 3D ($\circ$) lattice, the atoms are polarized along one axis of the array. The dashed black line is a guide to the eye representing the analytical scaling law $R_* \sim N^{\frac{3}{2} - \frac{1}{2D}} \Gamma_0$. Inset: comparison between the numerical approximation for the maximal decay rate R_* given by R_{SDP} and the largest eigenvalue of $\hat{H}_T$ for arrays of up to $N = 25$ atoms obtained by exact diagonalization. The dashed coloured lines indicate the upper and lower bounds, obtained by numerically evaluating equation (4).

j), $R_* = N(N+2)\Gamma_0/4$ (refs. 16,36). These two cases serve as trivial lower and upper bounds, respectively, for R_* in arbitrary environments. In general, for systems with non-negative dissipative couplings ($\Gamma_{ij} \geq 0 \forall i, j$), we find $R_* \sim \sum_{i,j} \Gamma_{ij}$ (Supplementary Section A.3). In what follows, however, we do not make such a restrictive assumption.

We establish bounds on R_* using product states. Although other methods exist, our approach based on product states provides an important physical insight: entanglement is not necessary for a system to dissipate at a rate near the theoretical maximum scaling. This finding complements previous observations on the role of entanglement in spontaneous transient superradiance³⁷. We stress that our use of product states here is fundamentally different from many previous analytical studies based on mean-field theory, since the state decaying at rate R_* is, in general, highly entangled.

We obtain the upper bound by harnessing well-established theoretical guarantees for product-state approximations¹⁵, and the lower bound from the variational product state ansatz $|\psi\rangle \propto \bigotimes_{j=1}^N (|g_j\rangle + e^{-i\phi_j}|e_j\rangle)$, where $\{\phi_j\}$ are locally dependent relative phases between $|e\rangle$ and $|g\rangle$ determined by the dominant eigenvector of Γ . This yields the general bounds (Methods) as

$$\max\left\{N\Gamma_0, \frac{N\Gamma_{\max}}{4(\Delta^2 + 1)}\right\} \leq R_* \leq \frac{N}{2}(3\Gamma_{\max} - \Gamma_0), \quad (4)$$

where $0 \leq \Delta \leq \sqrt{N-1}$ is the relative fluctuation of the entries of $\alpha^{(1)}$, the dominant eigenvector of Γ . More specifically, we define $\Delta = \sqrt{\text{Var}(|\alpha^{(1)}|)/\langle|\alpha^{(1)}|\rangle}$, where $\langle|\alpha^{(1)}|\rangle$ and $\text{Var}(|\alpha^{(1)}|)$ are the mean and variance of the absolute entries of $\alpha^{(1)}$, respectively. The lower bound is tighter if decay is delocalized (that is, if the brightest collective jump operator has approximately uniform spatial support over all qubits), characterized by $\Delta = O(1)$. In particular, for a translationally invariant system, $\Delta = 0$. For non-interacting qubits with identical decay rate $\Gamma_{ii} = \Gamma_0$ (such that $\Gamma_{\max} = \Gamma_0$ and $R_* = N\Gamma_0$), both bounds are saturated. The lower bound is saturated for large N in the Dicke limit, where $\Gamma_{\max} = N\Gamma_0$, $\Delta = 0$ and $R_* = N^2\Gamma_0/4 + O(N\Gamma_0)$. Equation (4) also implies that for ‘sufficiently weak’ interactions (such that $\Gamma_{\max}$ is asymptotically

independent of N), the maximal decay rate scales only linearly with system size. This generalizes some of the authors’ recent results on the impossibility of Dicke superradiance with nearest-neighbour interactions³⁴, to systems with arbitrary interaction range and geometry.

Universal scaling laws

Our bounds are tight for systems with delocalized decay (differing only by a constant factor), thereby yielding scaling laws for the maximal decay rate. Taking $\Delta = O(1)$ in equation (4), we find

$$R_* \sim N\Gamma_{\max}, \quad (5)$$

which is one of the main results of this paper. Despite its apparent simplicity, the scaling law $R_* \sim N\Gamma_{\max}$ in the delocalized regime is non-trivial and certainly not true for arbitrary systems. More broadly, in Supplementary Section A.5, we prove that there are no general scaling laws on R_* that depend solely on the system size and the spectrum of Γ , which we show by constructing explicit, though non-physical, mathematical counterexamples. Since $\Gamma_{\max}$ can be computed numerically in $O(N^3)$ time, the scaling law provides an efficient scheme to approximate R_* for large system sizes with quasi-translational invariance (that is, such that $\Delta = O(1)$). Equation (5) remains valid for the general master equation (1), even in the presence of local terms $\hat{H}_{\text{local}}$ and $\mathcal{D}_{\text{local}}$. As we show in the Methods, these terms only shift R_* by a subleading amount $O(N\Gamma_0)$. Physically, this implies that the largest decay rate is always dominated by collective dissipation. Moreover, as discussed in Supplementary Section A.1, the scaling law is also robust to disorder in Γ .

Our scaling law reveals important insights about the N^2 scaling in Dicke superradiance: one factor of N arises from the permutation symmetry, and the other one from the delocalized nature of the dominant decay channel together with a non-vanishing excitation density at large N . It may seem surprising that a product state yields the same asymptotic decay rate as the entangled Dicke state, but this can be thought of as an instance of the quantum de Finetti theorem³⁸. Since $\hat{H}_T$ is 2-local, it suffices to only consider the two-body reduced density matrix of the permutationally symmetric Dicke state, which is close to a product state (with trace distance vanishing as $1/N$). Our results show that

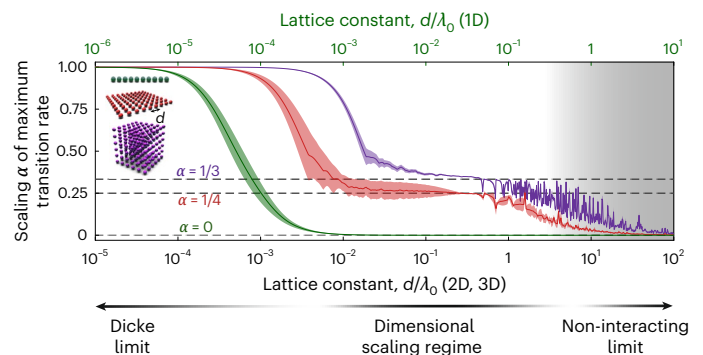


Fig. 3 | Finite-size effects in the scaling of the largest transition rate as a function of lattice constant. Data are obtained from a best fit to $\Gamma_{\max} = \beta N^n \Gamma_0$. These data are presented as fit values (solid lines) with the corresponding 1σ confidence interval (coloured regions). For small lattice constants, the scaling reaches the Dicke limit ($\alpha = 1$), and for very large lattice constants, the ensemble becomes non-interacting ($\alpha = 0$). For a region of physically relevant lattice constants, the fits agree with the dimensional scaling of equation (7), that is, $\alpha = 1/2 - 1/2D$. The atoms form a square lattice and are polarized parallel to one axis of the array. The fits are done with $n = \{7, 6, 9\}$ points equally distributed over a region $N_{\text{1D}} \in [2, N_{\text{1D}}^{\text{max}}]$, where $N_{\text{1D}}^{\text{max}} = \{30000, 250, 40\}$ for one, two and three dimensions, respectively. The grey area shows the region in which the fit is not accurate ($R^2 < 0.95$; Supplementary Section B.1). The top axis shows the lattice constant exclusively for 1D arrays. The dashed lines represent analytical scaling.

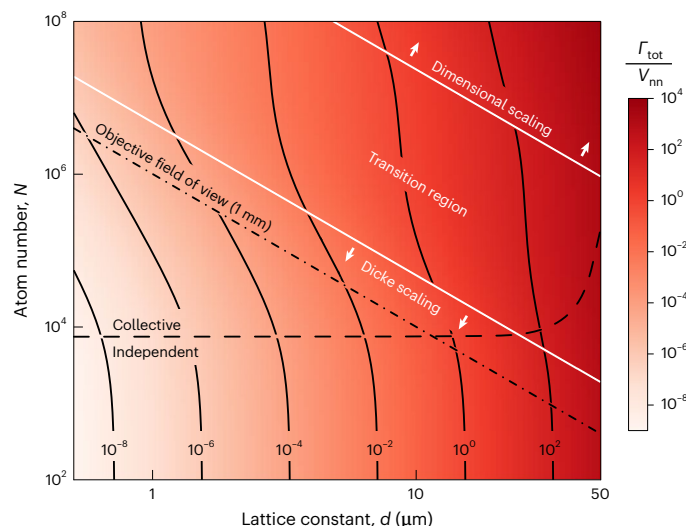


Fig. 4 | Implications of the scaling laws for quantum computing and simulation with Rydberg atom arrays. Ratio between the largest total leakage Γ_{tot} (defined in the main text) and the Rydberg–Rydberg interaction V_{nn} , for the $53S_{1/2}$ state in 2D Rydberg arrays of ^{87}Rb atoms. The solid white lines delimit the regions in which the collective decay rate follows Dicke (lower line) and dimensional (upper line) scalings. The transition region interpolates between the two regimes. Above the dashed line, collective decay exceeds the total independent leakage $\Gamma_{\text{sp}} + \Gamma_{\text{bb}}$ originating from spontaneous emission and black-body processes at $T = 300\text{ K}$ (Methods). The dash-dotted line indicates the state-of-the-art limit size, imposed by the objective’s field of view ($\sim 1\text{ mm}$).

the accuracy of the mean-field (product state) ansatz holds more generally, even when the permutation symmetry is broken.

Maximal decay rate of atomic arrays in free space

We now focus on ordered lattices of two-level atoms in free space, whose interactions are described by the propagator of the electromagnetic field evaluated at the resonance frequency ω_0 , which is a long-ranged function with oscillating sign (Supplementary Section B.1). This makes the problem of finding the ground state of $-\hat{H}_T$ non-trivial and is, thus, a perfect candidate to showcase the strength of our theoretical tools. Nevertheless, our formalism is not restricted to electric-dipole-mediated interactions in free space but can also describe magnetic-dipole or electric-quadrupole interactions in arbitrarily complex dielectric structures.

In the large- N limit, (and for a large range of lattice constants d , the functional dependence on system size of the largest transition rate is only determined by the dimensionality of the array³⁹). One can relate the scaling with N to the presence of divergences of $\Gamma(\mathbf{k})$ in reciprocal space as $|\mathbf{k}|$ approaches $k_0 \equiv \omega_0/c$ (c , the speed of light) by assuming that the single-excitation eigenstates of large arrays can be asymptotically well-approximated by spin waves^{40–43} and then performing a discrete sampling of the Brillouin zone³⁹. Divergences do not occur for one-dimensional (1D) arrays. They appear for two-dimensional (2D) and three-dimensional (3D) lattices, as the number of atoms per volume increases, enhancing the constructive interference of photon emission for certain wavevectors. For a D -dimensional array, the largest transition rate scales as

$$\Gamma_{\text{max}}^{(D)}/\Gamma_0 \sim (k_0 d)^{-\frac{D+1}{2}} N^{\frac{D-1}{2D}}. \quad (6)$$

This result can be easily generalized to compute the scaling of Γ_{max} for an arbitrary D -dimensional lattice in δ -dimensional free space, with $D \leq \delta$ (Supplementary Section B.2). For this scaling to determine $R_\star$ via equation (5), the jump operator associated with the largest transition rate must be delocalized. In three-dimensional free space ($\delta = 3$), this

is expected to hold. Although we explicitly verify this delocalization numerically for 1D and 2D arrays (Methods), an exact computation of the delocalization parameter Δ for large 3D arrays is numerically intractable. Nevertheless, as shown below, we independently validate the scaling law for all dimensions (including 3D) via semidefinite-program (SDP) relaxations.

The asymptotic scaling of the maximal decay rate depends on the array dimensionality ($D \in \{1, 2, 3\}$) as

$$\frac{R_\star^{(D)}}{\Gamma_0} \sim N^{\frac{3}{2} - \frac{1}{2D}}. \quad (7)$$

This dimensional scaling, a main result of the paper, is universal. It is independent of microscopic details (such as lattice constant, geometry and polarization), which only appear as prefactors. The scaling in equation (7) differs substantially from that expected in the Dicke limit as $N \rightarrow \infty$. The departure is the largest for 1D arrays, whose largest decay rate effectively scales as that of a collection of non-interacting atoms. In the Methods, we show that the upper bound on $R_\star$ remains robust against common experimental imperfections, such as position disorder and finite temperature, and provide numerical evidence indicating that the scaling of the lower bound is also preserved.

We benchmark our analytical scaling laws via a SDP relaxation⁴⁴, which yields a rigorous upper bound on $R_\star$ and, via approximation guarantees, a corresponding lower bound. SDP relaxations have been used to lower-bound different types of ground-state problems^{45–48} and, more recently, ground-state observables⁴⁹. The SDP relaxation is formulated as

$$\begin{aligned} R_{\text{SDP}} &= \max_{X \succeq 0} \quad \text{Tr}(\Gamma X) \\ \text{subject to} \quad & X_{ii} = 1 \quad \forall i = 1, \dots, N \end{aligned} \quad (8)$$

which can be solved in polynomial time. This is a relaxation since we can write $R = \text{Tr}(\Gamma M)$, where M is a positive matrix with elements $M_{ij} = \langle \hat{\sigma}_i^+ \hat{\sigma}_j^- \rangle$. Note that the specific form of M_{ij} imposes physical constraints on M . The SDP relaxation above omits these constraints and, thus, upper-bounds $R_\star$. Using standard techniques for establishing approximation guarantees of SDP relaxations⁵⁰, we show that R_{SDP} also provides a lower bound to $R_\star$ up to a constant factor (Methods).

This establishes

$$R_{\text{SDP}} \sim R_\star, \quad (9)$$

as a universal relation valid for arbitrary systems, regardless of whether decay is delocalized. Although R_{SDP} is generally not analytically tractable, this result indicates that SDP relaxations offer a powerful numerical framework for extracting empirical scaling laws of $R_\star$ at large system sizes, particularly for more complicated systems in which analytical approaches are unavailable. Using an SDP solver⁵¹, we obtain a good approximation of $R_\star$ (Fig. 2) for arrays with lattice constant $d = 0.4\lambda_0$, where $\lambda_0 = 2\pi/k_0$ is the wavelength associated with the resonant transition.

For large N , the numerical approximations to $R_\star$ given by the SDP follow the analytical scaling law of equation (7), thereby validating the dimensional scaling. For visualization purposes, we shift the dataset corresponding to each lattice dimension by a multiplicative factor (a constant upward shift in logarithmic scale). These shifts do not affect the scaling and highlight the excellent agreement between the numerical results and the analytical scaling laws. Since the dominant decay is delocalized, Fig. 2 validates our scaling law for $R_\star$. We also compare the SDP results (without shifts) with the exact diagonalization of $\hat{H}_T$ for up to $N = 25$ atoms (inset).

For sufficiently large atom numbers, the scalings hold regardless of the lattice constant. For finite N , however, they depend on both

lattice constant and lateral size $L = N^{1/d}$ (Fig. 3). As discussed in Supplementary Section B.1, by taking the limits of the expression for Γ in the appropriate order ($k_0 d \rightarrow 0$ before $N \rightarrow \infty$), we confirm that for $L \ll \lambda_0$, one recovers Dicke's scaling, that is, $\Gamma_{\max} = N\Gamma_0$. For arrays with a large lattice constant, $d \gg \lambda_0$, following a similar procedure yields the limit of non-interacting atoms, that is, $\Gamma_{\max} \simeq \Gamma_0$. These results indicate that asymptotic scaling is a robust bulk property, with boundary effects becoming negligible for sufficiently large arrays. We determine the crossover between 'non-interacting' and 'collective' behaviours by identifying the parameters for which there is an asymptotic change in the scaling of R_* , from linear to superlinear. For 2D and 3D arrays, the number of atoms required for such a crossover is $N^{(\text{crit})} \simeq \eta(k_0 d)^6$, where $\eta = 0.02$ and 5 , respectively. As expected, for large interparticle distances, the number of atoms required for R_* to be superlinear grows rapidly.

Experimental implications

The scaling laws impact a broad range of areas, including quantum optics, out-of-equilibrium phase transitions, quantum simulation and fault-tolerant quantum computation.

(1) Transient superradiance beyond the Dicke limit. Our findings on the scaling of R_* crucially address fundamental problems in quantum optics, such as transient superradiance in extended systems^{16,19,36}. For instance, they set a rigorous upper bound on the scaling of the superradiant burst. Although this upper bound may be violated if light is collected only over a small solid angle, new scaling laws can be derived taking into account the detector aperture. Furthermore, in Supplementary Section C.1, we reveal a connection between the early time correlations that determine the appearance of a superradiant burst¹⁹ (achieved under dynamical evolution) and R_* (which may be inaccessible by dynamics). Although our approach does not capture dynamical evolution, it allows us to predict that the timescale of a superradiant burst from an initially fully excited array of atoms in free space scales (up to a logarithmic factor) as $T_R \sim 1/\Gamma_{\max}$ (Supplementary Section B.3). This timescale imposes constraints on the array size required for retardation effects to be negligible, thereby ensuring the Markovian evolution assumed in equation (1).

(2) Superradiant lasing in free space. Superradiant lasing, in which incoherently pumped atoms spontaneously radiate coherent light, is known to occur in cavities^{1,2}. It is, however, unknown whether this phenomenon occurs in other environments. By means of our scaling laws, in Supplementary Section C.2, we derive an upper bound on the emitted intensity, $I \leq \hbar\omega_0 R_*$, and on the optimal pump rate, $W_* \leq 2R_*/N$. This upper bound is tight for the Dicke limit (that is, for atoms in a cavity), yielding $W_* = N\Gamma_0/2$. These results rule out superradiant lasing for 1D arrays in free space (as there is never a superlinear scaling for the intensity). They suggest that a superradiant lasing phase transition might occur for 2D and 3D arrays, with a scaling of the lasing region determined by equation (7).

(3) Driven-dissipative Dicke phase transition. Coherently pumped atoms in a cavity exhibit a second-order phase transition (at a critical pumping rate $N\Gamma_0/2$) from a magnetized phase characterized by a collective polarization to a paramagnetic phase^{3–5}. Beyond the well-studied case of atoms in a cavity, this phenomenon—also called collective resonance fluorescence—has remained largely unexplored until recently^{5,52–55}. Our scaling laws can be harnessed to readily show that R_* sets the scaling of both the threshold drive intensity ($\eta_c \sim \sqrt{\Gamma_0 R_*/N}$) and the emitted light intensity below the threshold ($I \sim \hbar\omega_0 R_*$) (Supplementary Section C.3). These analytical results are in agreement with recent numerical studies on collective resonance fluorescence of free-space arrays^{54,55}.

(4) Quantum simulators and processors with Rydberg atoms. For microwave transitions, such as between Rydberg states (where $\lambda_0 \approx 10$ mm), typical experimental implementations lie within the collective decay regime. Enhanced decay in dense Rydberg ensembles has

been attributed to Dicke-like superradiance⁵⁶. In Rydberg manifolds, collective decay may increase leakage error rates from the computational subspace in quantum processors, particularly in massively parallelized architectures with macroscopic Rydberg occupation. In quantum simulators, collective decay may become a relevant error source in large systems operating over long simulation times, potentially contributing to Rydberg avalanches whose microscopic origin remains unclear^{57,58}.

As an example, we estimate the potential impact of collective decay using the setup in ref. 59, which uses a 2D array of tweezer-trapped⁸⁷Rb atoms for quantum computing. A similar platform has also been used to simulate complex many-body quantum dynamics⁶⁰. The sources of leakage from the Rydberg state $53S_{1/2}$ include both collective ($\Gamma_{\text{col}} \equiv \Gamma_{\max}/4 \approx R_*/N$) and independent (that is, single-atom spontaneous emission, Γ_{sp}) decay to other levels, as well as black-body-induced transitions at a rate Γ_{bb} . The total leakage rate $\Gamma_{\text{tot}} = \Gamma_{\text{col}} + \Gamma_{\text{sp}} + \Gamma_{\text{bb}}$ is the difference between the (possibly black-body enhanced) transition rates from and to the $53S_{1/2}$ Rydberg level. Figure 4 shows the ratio between the total leakage rate Γ_{tot} and the coherent Rydberg–Rydberg interaction rate $V_{\text{nn}} = C_6/d^6$ between neighbouring atoms. The solid white lines in Fig. 4 mark the boundaries of the Dicke and dimensional scaling regimes (Methods and Supplementary Section C.4). In the transition region, the ratio $\Gamma_{\text{tot}}/V_{\text{nn}}$ is obtained by interpolating between the two asymptotic scalings. To estimate Γ_{col} , we adopt the prefactor derived from the product-state ansatz among the possible values consistent with the bounds in equation (4). This represents a realistic estimate, since such states are both physically well-motivated and experimentally accessible, and they reproduce R_* exactly in the Dicke limit. Similar considerations apply to other highly excited states, which can evolve—over a timescale Γ_{col}^{-1} —into configurations with leakage rates scaling as R_* through transient superradiance (see point (1) above). For simplicity, although the Rydberg state can decay via numerous channels, we consider only the transition with the largest $\Gamma_{\max}$ when evaluating Γ_{col} , as it dominates all others for large N (refs. 61,62).

Collective decay remains sub-dominant in current platforms of a few hundred atoms, but next-generation arrays hosting thousands of atoms⁶³ should enable the direct exploration of its detrimental effects. We estimate that a 40×200 array of Rydberg atom pairs (with $2\text{-}\mu\text{m}$ intrapair and $d = 12\text{ }\mu\text{m}$ interpair spacing) performing parallel gates as in ref. 59 would reach a gate error of $\varepsilon \approx 0.3\%$, with collective decay contributing one-third (0.1%) of the total error budget (Supplementary Section C.4). The results shown in Fig. 4 for 2D arrays also hold in three dimensions, as the ratio $\Gamma_{\text{tot}}/V_{\text{nn}}$ only slightly increases with dimensionality. This suggests that decoherence due to collective decay becomes more relevant in 3D arrays, where larger arrays can be built. Our analysis indicates that further investigation of collectively enhanced leakage errors in large-scale Rydberg atom arrays is critical to assess the scalability of these platforms for quantum computation and simulation purposes. In particular, the use of alternative atomic species or other Rydberg levels may allow for dimensional scalings (instead of the more detrimental Dicke scaling), thereby reducing collective effects in such platforms.

(5) Quantum error correction and typical states. The scaling law in equation (7) indicates that correlated decay may hamper quantum error correction^{7,64,65}, as the error rate per qubit scales (in the worst case) as $\sim R_*/N$, which grows with N in two dimensions and above. Correcting for these errors requires acting on a timescale that shortens with N . Nevertheless, in Supplementary Section A.6, we prove that the decay rate for typical stabilizer states is close to $N\Gamma_0/2$, implying that they do not experience correlated decay, due to random phases between qubits. This does not mean that the scaling laws for R_* are irrelevant in practice, since even simple states like the product state $|+\rangle^{\otimes N}$ may be superradiant. Understanding the effect of collective decay on specific quantum error correcting codes warrants further investigation.

Outlook

Our results highlight a class of universal scaling laws that govern the fundamental limits of decay in many-body quantum systems. These findings open several avenues for both experimental exploration and theoretical development. One important direction is quantum metrology, where correlated decay may impose fundamental constraints. Recent experiments on lattice clocks³⁰ and spin squeezing^{31–33} have investigated the role of Hamiltonian power-law dipole–dipole interactions. The dissipative counterpart of the interaction is typically neglected (as dephasing noise is currently the main source of error), although it sets a fundamental limit on the time available to generate and utilize metrologically useful states. Although the decay rate of typical (Haar-random) states is close to $N\Gamma_0/2$, quantum metrology protocols often rely on a carefully selected set of states (for example, Dicke states or squeezed states), which provide metrological advantage⁶⁶. Investigating the many-body decay of such states will prove essential to establish possible limitations on such schemes.

From a theoretical perspective, our approach illustrates the power of quantum approximation techniques to predict relevant properties of open many-body systems. Our formalism can be extended to yield upper bounds on the rates of change of higher-order observables such as k -point correlation functions (in this work, $\hat{n}_{\text{exc}}$ corresponds to $k=1$). Replacing $\hat{n}_{\text{exc}}$ in equation (2) with a k -local observable, the ‘auxiliary’ Hamiltonian $\hat{H}_F$ will, in general, be $(k+1)$ -local, and the optimal product state provides an approximation with a multiplicative error of at most 3^{k+1} , independent of system size⁴⁵. Furthermore, our approach could be adapted to study the steady-state behaviour of driven-dissipative systems and to predict the scaling of correlation functions or other physical observables⁴⁹. More advanced SDP relaxations such as the quantum Lasserre hierarchy⁴⁸ could yield tighter bounds for generic systems. These methods are not restricted to spin models, and could be extended to study ensembles of interacting fermions or bosons, as well as to disordered systems. Moreover, invoking time-reversal arguments, our scalings apply to the maximal absorption rate, which may have implications for quantum batteries and light harvesting protocols. Given their generality, we anticipate that these ideas will become a powerful tool to investigate universal properties of large-scale many-body open quantum systems.

Online content

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Methods

Derivation of lower bound on $R_\star$

By means of the variational principle and equation (3), we find

$$R_\star = \max_{|\psi\rangle} \sum_{\mu} \Gamma_{\mu} \langle \psi | \hat{c}_{\mu}^{\dagger} \hat{c}_{\mu} | \psi \rangle \geq \Gamma_{\max} \|\hat{c}_1^{\dagger} \hat{c}_1\|. \quad (10)$$

Substituting the definition of the collective operators $\hat{c}_{\mu}$, we arrive at

$$\begin{aligned} R_\star &\geq \Gamma_{\max} \left\| \sum_{i,j} \alpha_i^{(1)*} \alpha_j^{(1)} \hat{\sigma}_i^+ \hat{\sigma}_j^- \right\| \\ &= \Gamma_{\max} \left\| \sum_{i,j} |\alpha_i^{(1)}| |\alpha_j^{(1)}| \hat{\sigma}_i^+ \hat{\sigma}_j^- \right\| \\ &= \Gamma_{\max} \max_{|\psi\rangle} \sum_{i,j} |\alpha_i^{(1)}| |\alpha_j^{(1)}| \langle \psi | \hat{\sigma}_i^+ \hat{\sigma}_j^- | \psi \rangle \\ &\geq \frac{\Gamma_{\max}}{4} \left(\sum_i |\alpha_i^{(1)}| \right)^2, \end{aligned} \quad (11)$$

where we have absorbed the phases ϕ_i of $\alpha_i^{(1)}$ into the lowering operators $\hat{\sigma}_i^- = e^{i\phi_i} \hat{\sigma}_i^-$. The last inequality is obtained by choosing the product state $|\psi\rangle = \bigotimes_i (|g\rangle + \exp(-i\phi_i)|e\rangle)/\sqrt{2}$ such that $\langle \psi | \hat{\sigma}_i^+ \hat{\sigma}_j^- | \psi \rangle = 1/4$ if $i \neq j$ and $\langle \psi | \hat{\sigma}_i^+ \hat{\sigma}_i^- | \psi \rangle = 1/2$. The lower bound, thus, depends on how the collective operator $\hat{c}_1$ is spatially supported on the N qubits, quantified by the variance of $|\alpha_i^{(1)}|$, which reads

$$\begin{aligned} \text{Var}(|\alpha^{(1)}|) &= \frac{1}{N} \sum_i \left(|\alpha_i^{(1)}| - \overline{|\alpha^{(1)}|} \right)^2 \\ &= \frac{1}{N} - \frac{1}{N^2} \left(\sum_i |\alpha_i^{(1)}| \right)^2. \end{aligned} \quad (12)$$

The second line of the equation is obtained by using $\sum_i |\alpha_i^{(1)}|^2 = 1$ and $|\overline{|\alpha^{(1)}|}| = \sum_i |\alpha_i^{(1)}|/N$. Defining the relative fluctuation of $|\alpha_i^{(1)}|$ as $\Delta = \sqrt{\text{Var}(|\alpha^{(1)}|)/|\overline{|\alpha^{(1)}|}|}$ yields the lower bound

$$R_\star \geq \frac{N\Gamma_{\max}}{4(1+\Delta^2)}. \quad (13)$$

Explicitly,

$$\Delta^2 = \frac{N}{\left\| |\alpha^{(1)}| \right\|_1^2} - 1, \quad (14)$$

where $\alpha^{(1)} = (\alpha_1^{(1)} \dots \alpha_N^{(1)})^T$ is the dominant eigenvector of Γ , and $\|\alpha^{(1)}\|_p$ is the L^p vector norm. Using the inequality $\|\alpha^{(1)}\|_2 \leq \|\alpha^{(1)}\|_1 \leq \sqrt{N} \|\alpha^{(1)}\|_2$ and $\|\alpha^{(1)}\|_2 = 1$ by normalization, we have the bound

$$0 \leq \Delta \leq \sqrt{N-1}. \quad (15)$$

The upper bound $\Delta = \sqrt{N-1}$ is attained for independent emitters. Intuitively, Δ quantifies the spatial uniformity of the brightest collective jump operator $\hat{c}_1$. Thus, we define the delocalized decay regime to be $\Delta = O(1)$. For translationally invariant systems, $|\alpha_i^{(\mu)}| = N^{-1/2}$ for all $i = 1 \dots N$ and $\mu = 1 \dots N$, such that $\Delta = 0$.

Derivation of upper bound on $R_\star$

Let us write $\hat{H}_\Gamma = \hat{H}_{\text{diag}} + \hat{H}_{\text{XY}}$, where $\hat{H}_{\text{diag}} = \hbar \sum_{i=1}^N \Gamma_{ii} \hat{\sigma}_i^+ \hat{\sigma}_i^-$ and

$$\hat{H}_{\text{XY}} = \frac{\hbar}{4} \sum_{i \neq j} \Gamma_{ij} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y). \quad (16)$$

By the triangle inequality, $R_\star \leq N\Gamma_0 + \|\hat{H}_{\text{XY}}\|/\hbar$. Since $\hat{H}_{\text{XY}}$ is 2-local and traceless, we use a recent result from ref. 15 to write

$$\|\hat{H}_{\text{XY}}\| \leq 6\hbar R_{\text{prod}}(\hat{H}_{\text{XY}}), \quad (17)$$

where $R_{\text{prod}}(\hat{H}_{\text{XY}})$ is the best product-state approximation to $\|\hat{H}_{\text{XY}}\|/\hbar$. Restricting to product states, $\hat{H}_{\text{XY}}$ reduces to a classical XY Hamiltonian

$$H_{\text{XY}} = \frac{\hbar}{4} \sum_{i \neq j} \Gamma_{ij} \mathbf{s}_i \cdot \mathbf{s}_j = \frac{\hbar}{4} \text{Tr}(\tilde{\Gamma} \Sigma), \quad (18)$$

where $(\Sigma)_{ij} = \mathbf{s}_i \cdot \mathbf{s}_j$ is the Gram matrix for the unit vectors $\{\mathbf{s}_i \in \mathbb{R}^2\}$ and $\tilde{\Gamma}$ is the off-diagonal matrix $\Gamma - \Gamma_0 I_N$.

Using the inequality $\text{Tr}(\tilde{\Gamma} \Sigma) \leq \|\tilde{\Gamma}\| \text{Tr}(\Sigma)$ (ref. 67), we obtain

$$R_{\text{prod}}(\hat{H}_{\text{XY}}) \leq \frac{N}{4} (\Gamma_{\max} - \Gamma_0). \quad (19)$$

Thus, by invoking equation (17), we obtain the upper bound

$$R_\star \leq N\Gamma_0 + \frac{3N}{2} (\Gamma_{\max} - \Gamma_0) = \frac{N}{2} (3\Gamma_{\max} - \Gamma_0). \quad (20)$$

Effect of local Hamiltonian and dissipation on $R_\star$

Here we show that $\hat{H}_{\text{local}}$ and $\mathcal{D}_{\text{local}}$ in equation (1) contribute only an $O(N\Gamma_0)$ shift to $R_\star$ and, therefore, can be omitted without affecting the scaling law. A more formal derivation is provided in Supplementary Section A.1. We assume that each qubit in the system interacts with $O(1)$ other qubits via $\hat{H}_{\text{local}}$ and $\mathcal{D}_{\text{local}}$. Here

$$\hat{H}_{\text{local}} = \hbar \sum_j \eta_j \hat{\rho}_j \quad (21)$$

and

$$\mathcal{D}_{\text{local}}(\hat{\rho}) = \sum_j \kappa_j (\hat{\rho}_j \hat{\rho} \hat{\rho}_j - \hat{\rho}), \quad (22)$$

where $\hat{\rho}_j$ are spatially local Pauli operators with arbitrary coefficients $\{\eta_j, \kappa_j\}$ of order unity, in units of Γ_0 . There are $O(N)$ such Pauli operators, by locality. From equation (2), the contribution of $\hat{H}_{\text{local}}$ to the decay rate is

$$R = -\frac{i}{2} \sum_j \sum_{i=1}^N \eta_j \langle [\hat{\rho}_j, \hat{\sigma}_i^z] \rangle. \quad (23)$$

Since each $\hat{\rho}_j$ is supported only on a constant number of qubits, the commutator $[\hat{\rho}_j, \hat{\sigma}_i^z]$ evaluates to a linear combination comprising a constant number of Pauli operators, each with expectation value at most 1. Thus,

$$R \leq \text{const.} \times \sum_j |\eta_j| = O(N\Gamma_0). \quad (24)$$

Next, the contribution of $\mathcal{D}_{\text{local}}$ to R is

$$R = \sum_j \sum_{i=1}^N \frac{\kappa_j}{2} (\langle \hat{\sigma}_i^z \rangle - \langle \hat{\rho}_j \hat{\sigma}_i^z \hat{\rho}_j \rangle). \quad (25)$$

The term in the parentheses has a non-vanishing contribution of $O(1)$ if qubit i is in the support of $\hat{\rho}_j$. Thus, by the same counting argument as before, we get

$$R \leq \text{const.} \times \sum_j \kappa_j = O(N\Gamma_0). \quad (26)$$

In other words, the local Hamiltonian and dissipation can only contribute $O(N\Gamma_0)$ to the decay rate of any state $\hat{\rho}$ and, therefore, also $R_\star$.

Approximation guarantees of the SDP relaxation

R_{SDP} , defined in equation (8), is an upper bound on $R_\star$ by construction. Here we show that R_{SDP} also provides a lower bound to $R_\star$ up to a constant factor, thereby establishing the universal relation $R_{\text{SDP}} \sim R_\star$. The proof closely follows the seminal work in ref. 50 for the MAXCUT

problem. The main idea behind our proof is to use randomized rounding to construct a product state whose decay rate is guaranteed to approximate $R_\star$ up to a constant factor.

Let us denote the optimal solution of the SDP relaxation in equation (8) by the unit vectors $\{\mathbf{v}_i \in \mathbb{R}^N\}$, whose Gram matrix $X_\star$ solves the maximization problem. Next, we round the solution. We sample a random vector $\mathbf{g} \in \mathbb{R}^N$ with independent and identically distributed entries drawn from the Gaussian distribution, and define $b_i = \text{sign}(\mathbf{v}_i \cdot \mathbf{g}) \in \pm 1$. Note that this is equivalent to sampling a random hyperplane in $\mathbb{R}^N$ passing through the origin and assigning the binary variable b_i depending on which side of the hyperplane $\mathbf{v}_i$ lies in. This gives the Gram matrix B with matrix elements $B_{ij} = b_i b_j$. Averaging over the random vector $\mathbf{g}$, it can be shown that⁵⁰

$$\mathbb{E}[B_{ij}] = \mathbb{E}[b_i b_j] = \frac{2}{\pi} \sin^{-1}(\mathbf{v}_i \cdot \mathbf{v}_j). \quad (27)$$

Intuitively, this tells us that if $\mathbf{v}_i$ and $\mathbf{v}_j$ have high overlap, the rounded solution b_i and b_j will be preferentially aligned on average. Now, by observing that $\mathbb{E}[B] \geq (2/\pi)X_\star$, which can be derived using the Schur product theorem and the Taylor expansion of $\sin^{-1}(x)$, we obtain

$$\frac{2}{\pi} R_{\text{SDP}} \leq \mathbb{E}[R_{\text{round}}], \quad (28)$$

where $R_{\text{round}} = \text{Tr}(FB)$ is the cost function of equation (8) evaluated at $X = B$.

For each b_i , we can construct a single-qubit state as

$$|b_i\rangle = \frac{1}{\sqrt{2}}(|g\rangle + b_i |e\rangle), \quad (29)$$

which has $\langle \hat{\sigma}_i^- \rangle = b_i/2$. Then, defining the product state as $|\mathbf{b}\rangle = \bigotimes_{i=1}^N |b_i\rangle$, we can write

$$R_{\text{round}} = 4\langle \mathbf{b} | \hat{H}_F | \mathbf{b} \rangle + O(N\Gamma_0). \quad (30)$$

In other words, R_{round} is four times the decay rate of the product state $|\mathbf{b}\rangle$, up to an additive error of $O(N\Gamma_0)$, which we omit for simplicity (since it does not contribute asymptotically to the scaling laws). Thus, $R_{\text{round}} \leq 4R_\star$, resulting in $\mathbb{E}[R_{\text{round}}] \leq 4R_\star \leq 4R_{\text{SDP}}$, where we used the fact that $R_\star \leq R_{\text{SDP}}$.

Putting the above results together, we obtain asymptotically tight bounds on $R_\star$ in terms of R_{SDP} as

$$\frac{1}{2\pi} R_{\text{SDP}} \leq R_\star \leq R_{\text{SDP}}. \quad (31)$$

This implies that R_{SDP} follows the same scaling law as $R_\star$, that is, $R_{\text{SDP}} \sim R_\star$. The additive error $O(N\Gamma_0)$, which is suppressed in equation (31), does not affect the scaling laws.

Derivation of Γ_{max} scaling for free-space atomic arrays

For atomic arrays in three-dimensional free space, the dissipative matrix reads

$$\Gamma_{ij} = \frac{6\pi\Gamma_0}{k_0} \hat{\phi}^* \cdot \text{Im}\mathbf{G}(\mathbf{r}_i - \mathbf{r}_j, \omega_0) \cdot \hat{\phi}, \quad (32)$$

where $\hat{\phi}$ denotes the orientation of the transition matrix element. The electromagnetic Green's tensor is given by

$$\begin{aligned} \mathbf{G}(\mathbf{r}, \omega_0) \equiv & \frac{e^{ik_0 r}}{4\pi k_0^2 r^3} \left[(k_0^2 r^2 + ik_0 r - 1)\mathbb{1} \right. \\ & \left. + (-k_0^2 r^2 - 3ik_0 r + 3) \frac{\mathbf{r} \otimes \mathbf{r}}{r^2} \right], \end{aligned} \quad (33)$$

where $r = |\mathbf{r}|$.

To analytically determine the scaling of $\Gamma_{\text{max}} = \beta N^\alpha \Gamma_0$, we study divergences of the transition rates in reciprocal space, that is, $\Gamma(\mathbf{k}) = \sum_{ij} \Gamma_{ij} e^{-i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} / N$. Our approach is to analyse an infinite array and then relate its behaviour to that of large, finite systems. This is justified because in the large-particle limit, the collective jump operators are well-approximated by Fourier modes^{40,43,68}. We present here the analysis for a 2D array, and discuss other dimensionalities in the Supplementary Information. Assuming a 2D array in the x - y plane with atoms polarized perpendicular to the array, the transition rate of a spin wave with momentum $\mathbf{k} = (k_x, k_y)$ reads⁴³

$$\frac{\Gamma_{2D}^\perp(\mathbf{k})}{\Gamma_0} = \frac{3\pi}{k_0^3 d^2} \sum_{\mathbf{g}} \frac{|\mathbf{k} + \mathbf{g}|^2}{\sqrt{k_0^2 - |\mathbf{k} + \mathbf{g}|^2}}. \quad (34)$$

The sum extends over all reciprocal lattice vectors, $\mathbf{g} = 2\pi(n_x, n_y)/d$ for $n_x, n_y \in \mathbb{Z}$ that satisfy the condition $|\mathbf{k} + \mathbf{g}| < k_0$. Equation (34) diverges for $|\mathbf{k} + \mathbf{g}| \rightarrow k_0$. To show how this divergence translates into the asymptotic scaling $\Gamma_{\text{max}}/\Gamma_0 \sim N^{1/4}$, we choose $d < \lambda_0/2$, where $k_0 < \pi/d$, and the only term that contributes to the sum in equation (34) is $\mathbf{g} = 0$. Sampling $\Gamma_{2D}^\perp(\mathbf{k})$ on a finite $\sqrt{N} \times \sqrt{N}$ grid in momentum space, the wavevectors closest to the divergence are such that $|\mathbf{k}| = k_0(1 - \varepsilon)$, where $\varepsilon \leq 2\pi/k_0 d(\sqrt{N} + 1)$. Plugging this condition into equation (34) and expanding in orders of $\varepsilon \approx \lambda_0/L$, we readily find

$$\begin{aligned} \frac{\Gamma_{\text{max}}^\perp}{\Gamma_0} \simeq & \frac{3\pi}{\sqrt{2}(k_0 d)^2} \left[\left(\frac{L}{\lambda_0} \right)^{\frac{1}{2}} - \frac{7}{4} \left(\frac{\lambda_0}{L} \right)^{\frac{1}{2}} + \frac{19}{32} \left(\frac{\lambda_0}{L} \right)^{\frac{3}{2}} \right] \\ & + O(\lambda_0^{5/2}/L^{5/2}), \end{aligned} \quad (35)$$

so that the first-order contribution scales with $\alpha^{(2D)} = 1/4$ and $\beta^{(2D)} = 3\sqrt{\pi}/2(k_0 d)^{3/2}$ as long as $\varepsilon \ll 1$ ($L \gg \lambda_0$), as mentioned in the main text.

In the Dicke regime ($L \ll \lambda_0$) one needs to take the limit $L \rightarrow 0$ before resumming the Fourier transform. By expanding in orders of L/λ_0 and then resumming, we find

$$\begin{aligned} \frac{\Gamma_{\text{max}}^\perp}{\Gamma_0} = & N \left(1 - \frac{4\pi^2 L^2}{15\lambda_0^2} + \frac{17\pi^4 L^4}{525\lambda_0^4} \right) + \frac{4\pi^2 L^2}{15\lambda_0^2} \\ & - \frac{40\pi^4 L^4}{525\lambda_0^4} + \frac{23\pi^6 L^6}{525N\lambda_0^6} + O\left(\frac{L^6}{\lambda_0^6}\right), \end{aligned} \quad (36)$$

so that the first-order contribution scales with $\alpha^{(2D)} = 1$ and $\beta^{(2D)} = 1$ as long as $L \ll \lambda_0$. On the contrary, by taking the large separation limit first in equation (32), one recovers the limit of independent atoms $\Gamma_{2D}^\perp(\mathbf{k}) = \Gamma_0$ regardless of N , so that $\alpha^{(2D)} = 0$ and $\beta^{(2D)} = 1$. The transition between the independent and array regimes can be seen directly from equation (35), as the first term is larger than 1 (that is, $\Gamma_{\text{max}} > \Gamma_0$) for $N \geq 16(k_0 d)^6/81\pi^2$.

Generalizations of these results to the array regime for $d > \lambda_0/2$, the case with parallel polarization, as well as 1D and 3D arrays, follow analogously, and are discussed in detail in Supplementary Section B.1.

Delocalized decay in finite atomic arrays

For the scaling law $R_\star \sim N\Gamma_{\text{max}}$ to hold for atomic arrays, it is necessary for the brightest jump operator $\hat{c}_i$ to be delocalized such that $\Delta \sim O(1)$. As demonstrated in ref. 40, the collective jump operators of ordered atomic arrays are well-approximated by Fourier modes with quasi-momentum $\mathbf{k}$ in the limit of $N \gg 1$ and are, therefore, delocalized. We confirm this in Extended Data Fig. 1a,b (top), where we show that the Fourier transform of the brightest collective jump operator is localized (in momentum space) for both 1D and 2D arrays.

Specifically, we numerically obtain the principal eigenvector $\alpha^{(1)} = (\alpha_1^{(1)}, \dots, \alpha_N^{(1)})^T$ of Γ and compute its momentum-space distribution $|\alpha_{\mathbf{k}}^{(1)}|^2$, where $\alpha_{\mathbf{k}}^{(1)} = \sum_j^N \exp(i\mathbf{k} \cdot \mathbf{r}_j) \alpha_j^{(1)} / \sqrt{N}$. Furthermore, we show that the

width of $|\alpha_k^{(1)}|^2$, quantified through the variance Δk^2 , decreases with the size of the array (Extended Data Fig. 1a,b, bottom). These results are obtained for an array polarized along one of the axis of the array. For perpendicular polarization, a similar narrowing of the distribution in k space is observed, although with some oscillation modulating the decreasing uncertainty Δk .

Finally, to confirm that the upper and lower bounds to R_* are asymptotically tight, we compute Δ for a 2D array (Extended Data Fig. 1c). The values of Δ are seen to saturate to a finite constant value as N increases. This is confirmed in Extended Data Fig. 1d, where for a few values of d in the relevant regime ($0.1 \leq d/\lambda_0 \leq 0.5$), Δ saturates to a constant value for increasing system sizes.

For 3D arrays, we expect a similar behaviour. However, due to computational constraints on N , we are unable to provide numerical evidence to support this expectation.

Role of position disorder on the scaling laws for atomic arrays

Here we analyse the robustness of the scaling law for arrays, that is, equation (7), to displacement in atomic positions. Position disorder occurs as either imperfections in the position of the trap (for example, in tweezer arrays) or temperature fluctuations of the atomic centre-of-mass position (for example, in optical lattices). In both cases, R_* is obtained from equation (3), where Γ is replaced by Γ' computed by sampling equation (32) according to atoms' position probability distribution. We consider a Gaussian distribution centred around the lattice sites with a standard deviation $\sigma = \eta d$. If η is kept constant, σ corresponds to different values of the temperature, or of the tweezers' displacement depending on d . Different experiments use different atoms and atomic transitions, resulting in a wide range of values for λ_0 and, hence, d . In the following, we fix $\eta = 0.05$, which corresponds to typical values reported in recent experiments^{63,69,70}.

We study the effects of disorder on the upper and lower bounds to R_* in equation (4) separately.

The upper bound is entirely determined by the scaling of $\Gamma_{\max}$. In the limit of weak position disorder ($\eta \ll 1$), the largest eigenvalues of Γ' can be analytically computed using perturbation theory. After averaging over the atom position distribution, we obtain (appendix C of ref. 71)

$$\bar{\Gamma}'_{\max} \simeq (1 - \eta^2)\Gamma_{\max} + \eta^2\Gamma_0. \quad (37)$$

For 1D arrays, the robustness of the upper bound is sufficient to conclude the validity of the scaling law in equation (7) because a lower bound to the decay rate is always given by $N\Gamma_0$, which is trivially achieved by $|e\rangle^{\otimes N}$. For 2D and 3D arrays, the lower bound depends on the scaling of $\Gamma_{\max}$ and of the principal eigenvector of Γ through Δ (equation (4)). We numerically show that $\Delta \sim O(1)$ for the case of 2D arrays. This is done in Extended Data Fig. 2a, where we plot Δ as a function of d for different array sizes. For fixed d , Δ converges to a constant value as N increases (Extended Data Fig. 2b). The numerical results shown in Extended Data Fig. 2 together with the robustness of the scaling of $\Gamma_{\max}$ allows us to conclude that lower and upper bounds are asymptotically tight, leading to the scaling law in equation (7) for 2D arrays. For 3D arrays, we expect the scaling law to hold, but computing Δ for large arrays is challenging and we do not have sufficient numerical evidence to support this expectation.

Collective decay in Rydberg arrays

Atoms excited to the $5S_{1/2}$ level can leak to all levels $nP_{1/2}$ and $nP_{3/2}$, with $n \geq 5$. Out of all possible decay paths, R_* is always larger for the $5S_{1/2} \rightarrow 52P_{3/2}$ transition due to its long-wavelength nature. We simplify the multilevel nature of the system by assuming that for large atom numbers, the dominant collective channel will dominate over the rest^{61,62}, so that $\Gamma_{\text{col}} = \Gamma_{\max}^{5S_{1/2} \rightarrow 52P_{3/2}}/4$. All other transitions are assumed to contribute only to the spontaneous (Γ_{sp}) and black-body (Γ_{bb}) leakage.

The collective contribution to leakage (Γ_{col}) is obtained from the lower bound in equation (4) by assuming $\Delta = 0$. To estimate this quantity for large systems, we use the expansions in equations (35) and (36) when the expansion error ϵ is below 0.05. The error ϵ is defined as the contribution of the higher-order term in the expansions. The remaining region around $L \approx \lambda_0$ is calculated via a weighted smooth interpolation using the thin-plate spline method⁷². We look for a surface $\Gamma_{\text{col}}(d, N)$ that minimizes the function

$$\sum_i w_i (\Gamma_{\text{col}} - \Gamma_{\max}(d_i, N_i)/4)^2 + \mu \iint \|\nabla^2 \Gamma_{\text{col}}\|, \quad (38)$$

where we take as input a 200×200 grid of points d_i, N_i with $\Gamma_{\max}(d_i, N_i)$ given by the expansions, weights $w_i = 1 - \epsilon_i^4$, where ϵ_i is given by the expansion error ϵ evaluated at each point i , and smoothness parameter $\mu = 0.1$. The grid is taken over a large parameter space, $[0.05, 500] \mu\text{m} \times [1, 10^{10}]$ for Fig. 4 to minimize edge effects.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request. Source data are provided with this paper.

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Author contributions

All authors developed the theory. E.S., A.P., C.C.R. and W.-K.M. performed the calculations. All authors contributed to the discussions and to the writing of the paper.

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Competing interests

J.P. is a shareholder of Oratomic, Inc., a company developing fault-tolerant quantum computing technologies. The other authors declare no competing interests.

Additional information

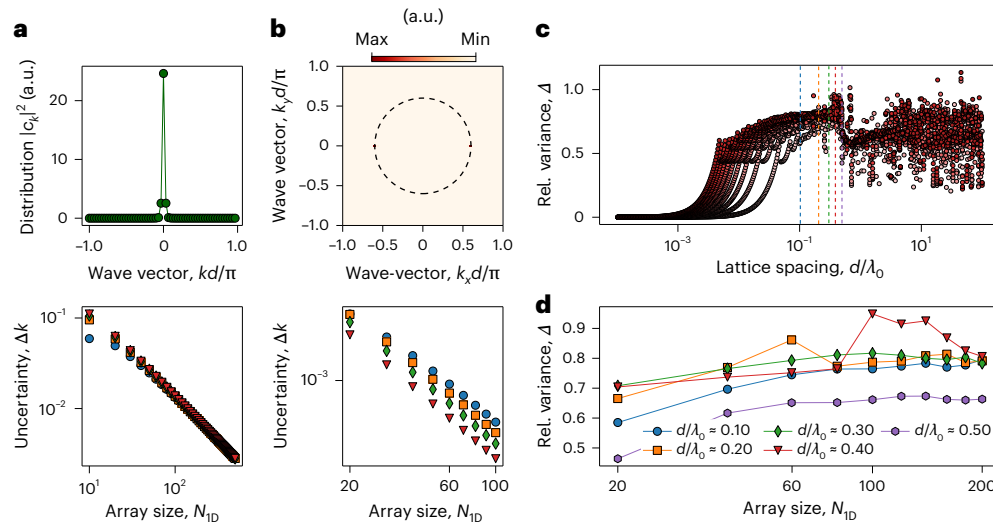
Extended data is available for this paper at <https://doi.org/10.1038/s41567-026-03448-4>.

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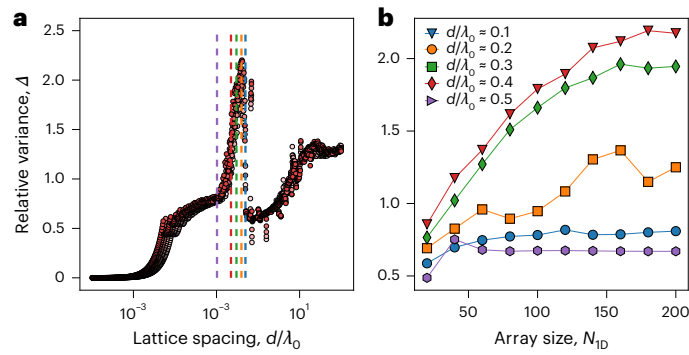
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Extended Data Fig. 1 | Delocalization of the brightest jump operator for 1D and 2D arrays. **a**, 1D arrays: (top) k -space distribution $|\alpha_k^{(1)}|^2$ of the brightest jump operator (principal eigenvector of I) for 60 atoms with $d = 0.3\lambda_0$ and (bottom) its variance Δk as a function of the array size N_{1D} . **b**, 2D arrays: (top) k -space distribution $|\alpha_{\mathbf{k}}^{(1)}|^2$ of the brightest jump operators for 60×60 atoms with $d = 0.3\lambda_0$ and (bottom) the square root of the determinant of its covariance matrix

as a function of the array size N_{1D} . **c**, Plot of the relative variance Δ of the principal eigenvector as a function of the lattice spacing for N_{1D} from 20 to 200 in steps of 20 (brightest to darkest markers). **d**, Scaling of Δ as a function of N_{1D} . Different markers correspond to different values of d/λ_0 , as specified in **d**. In all plots, atoms are polarized along the array axis.



Extended Data Fig. 2 | Effects of position disorder on R_* for arrays of atoms normally distributed around the sites of a square lattice with standard deviation $\sigma = 0.05d$. Atoms are polarized along one of the array axis. All results are obtained by averaging over 100 position realizations. **a**, Relative variance Δ of the principal eigenvector of f for a 2D array as a function of the lattice spacing

d/λ_0 and for five different values of N_{ID} starting from $N_{ID} = 100$ (lighter colour) and increasing in steps of 20 up to $N_{ID} = 180$ (darker colour). **b**, Scaling of Δ with N_{ID} for the values of d/λ_0 corresponding to the vertical dashed line of the same colour in **b**. The exact value of the lattice spacing for each line is specified in the legend.